A Banach algebra with a unique C^*-involution.

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Every C^*-algebra has this property. Indeed if A is a untial Banach algebra which is C^*-algebra with respect to involutions $*$ and $\#$, then if $x = x^*$ and f be a state on A (i.e., by [K&R1, Theorem 4.3.2] is a bounded linear functional satisfying $\|f\| = f(1) = 1$) then, $f(x) = \overline{f(x^*)} = \overline{f(x)}$, so that $f(i(x - x^{\#})) = i(f(x) - \overline{f(x)}) = 0$. Therefore, by [K&R1, Proposition 4.3.3] $sp(i(x - x^{\#})) = \{0\}$. Hence $i(x - x^{\#}) = 0$, by [K&R1, Proposition 4.1.1.(i)]. So $x^{\#} = x = x^*$. For an arbitrary element x with the real and imaginary parts x_1 and x_2, we have $x^* = x_1 - ix_2 = x^{\#}$. (If A doesn't have a unit, it is enough to consider its unitization).

Ref.

[**K&R1**] R.V. Kadilon, J.R. Ringrase, fundamentals of the theory of operator algebras (I), Acad. Press, 1983.