A closed subalgebra of a $C^*$-algebra that isn’t self-adjoint.

The disc algebra $\mathcal{A}(\Delta)$\footnote{Let \(\Delta\) denote the closed unit disc \(\{z \in \mathbb{C} \mid |z| \leq 1\}\). Suppose that \(\mathcal{A}(\Delta)\) denoted the set of all elements of \(C(\Delta)\) which are analytic on the interior of \(\Delta\). \(\mathcal{A}(\Delta)\) is a closed subalgebra of \(C(\Delta)\).} is a closed subalgebra of the $C^*$-algebra $C(\Delta)$. If $f$ and $\bar{f}$ both belong to $\mathcal{A}(\Delta)$, then by the Cauchy-Riemann equations $f$ will be constant. So $\mathcal{A}(\Delta)$ isn’t self-adjoint.