A closed left ideal of a $C^*$-algebra without any left approximate identity.

If $\xi$ is a unit vector in a Hilbert space $H$ with dimension at least 2, then $\Delta = \{ T \in B(H); T\xi = 0 \}$ is a closed left ideal in the $C^*$-algebra $B(H)$. If $\Delta$ has a left approximate identity $\{S_\alpha\}$ and $\eta \neq 0$ is a vector in $H$ such that $\langle \xi, \eta \rangle = 0$, then $\xi \otimes \eta \in \Delta$ and so $\lim_\alpha S_\alpha(\xi \otimes \eta) = \xi \otimes \eta$. Thus $\lim_\alpha \| (S_\alpha \xi - \xi) \otimes \eta \| = \lim_\alpha \| S_\alpha \xi - \xi \| \| \eta \| = 0$, hence $0 = \lim_\alpha \| S_\alpha \xi - \xi \| = \| \xi \|$, a contradiction. Thus $\Delta$ has no left approximate identity.

Note that for $\zeta_1$ and $\zeta_2$ in $H$ the rank one operator $\zeta_1 \otimes \zeta_2$ is defined by

$$ (\zeta_1 \otimes \zeta_2)(\zeta_3) = \langle \zeta_3, \zeta_2 \rangle \zeta_1. $$