An operator of index zero which isn't invertible.

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Let P be a non-trivial finite-rank idempotent in $B(X)$ (X is a Banach space), then $I - P$, the difference of an invertible operator and a compact operator, is Fredholm, of index $ind(I) = 0$, and non-invertible.