A bounded increasing sequence of self-adjoint operators on a Hilbert space which is not uniformly convergent.

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Assuming $(\xi_n)_{n\in\mathcal{N}}$ as an orthonormal basis for a separable infinite dimensional Hilbert space H, say l^2. Denote the linear span of $\{\xi_1, \xi_2, \cdots, \xi_n\}$ by Y_n. Let P_n be the projection onto the closed subspace Y_n. If $m < n$, then $Y_m \subset Y_n$ and so $0 < P_m < P_n$. Moreover $P_n - P_m$ is a projection and so $\| P_n - P_m \| = 1$ whenever $n \neq m$. Therefore (P_n) is an increasing sequence of self-adjoint operators which is not even a Cauchy sequence in uniform topology on $B(H)$.