Given a compact subset $K$ of $\mathcal{C}$, there exists a bounded operator $T$ on a Hilbert space such that $sp(T) = K$ and the set of eigenvalues of $T$ is dense in $K$.

Suppose that $H = l^2$, $(e_n)$ is the standard orthonormal basis for $H$ and $(\lambda_n)$ is a dense sequence in $K$. Set $T(\sum_{n=1}^{\infty} \alpha_n e_n) = \sum_{n=1}^{\infty} \lambda_n \alpha_n e_n$ where $(\alpha_n) \in l^2$. Obviously $K \subset sp(T)$. If $\lambda \notin K$, then $\inf\{|\lambda - \mu|; \mu \in K\} > 0$ and so $S(\sum_{n=1}^{\infty} \alpha_n e_n) = \sum_{n=1}^{\infty} (\lambda - \lambda_n)^{-1} \alpha_n e_n$ is a well-defined operator on $H$. $S$ is the inverse of $\lambda I - T$. Therefore $\lambda \notin sp(T)$. Thus $K = sp(T)$.

For every $n$, $Te_n = \lambda_n e_n$. In fact $\{\lambda_1, \lambda_2, \cdots\}$ is the set of all eigenvalues of $T$ that is dense in $sp(T)$. 

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