(a) A Banach space X and an operator $T \in B(X)$ having no nontrivial invariant subspace.
(b) A Banach space X and an operator $T \in B(X)$ having a nontrivial invariant subspace.

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(a) C.J. Read showed that if $X = l^1$ then there exists a bounded operator on l^1 having no nontrivial invariant subspace.

(cf. [C.J. Read, A solution to the invariant subspace problem, Bull. London Math. Soc., 16(1984), 337-401.]

(b) If $X = \mathcal{C}^n (n > 1), T \in B(\mathcal{C}^n) - \mathcal{C}I$ is an arbitrary operator and $\alpha \in \mathcal{C}$ is an eigenvalue of T, then $M = Ker(T - \alpha I)$ is a nontrivial subspace of X and $TM \subseteq M$.(I is the identity operator on $\mathcal{C}^n$)