(a) An injective operator on a Hilbert space H such that the range of $T, R(T)$, isn't dense in H.
(b) An operator S such that S is surjective but $Ker(S) \neq \{0\}$.

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Let H be a separable Hilbert space with the standard orthonormal basis (e_n).
(a) The unilateral shift operator $T(\alpha_1, \alpha_2, \cdots) = (0, \alpha_1, \alpha_2, \cdots)$ on H is injective and the closure of its range is the closed linear span $\{e_2, e_3, \cdots\}$ which doesn't contain e_1.

(b) If $S = T^*$, then $S(\alpha_1, \alpha_2, \cdots) = (\alpha_2, \alpha_3, \cdots)$. So S is surjective but $Ker(S) \neq \{0\}$, since it is the linear span of e_1.