A compact operator with no eigenvalues.

Let $X = C([0,1])$, $v : X \rightarrow X$ be the Volterra operator $v(f)(x) = \int_0^x f(t)dt$. If $S$ is the closed unit ball of $X$, then $v(S)$ is equicontinuous and pointwise-bounded, hence by the Arzela-Ascoli theorem, $v$ is compact. If for some $\lambda \in \mathcal{C}$ and $f \neq 0$ in $X$, $vf = \lambda f$, then $f(x) = \lambda f^\prime(x)$. So $\lambda \neq 0$ and $lnf(x) = \frac{x}{\lambda} + c$ for some $c \in \mathcal{R}$. Hence $f(x) = f(0)e^{\lambda x}$, $0e^{\lambda x} = 0$, a contradiction. $v$ has then no eigenvalue.