A compact operator with no eigenvalues.

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Let $X = C([0,1])$, $v : X \longrightarrow X$ be the Volterra operator $v(f)(x) = \int_0^x f(t)dt$. If S is the closed unit ball of X, then $v(S)$ is equicontinuous and pointwise-bounded, hence by the Arzela-Ascoli theorem, v is compact. If for some $\lambda \in \mathcal{C}$ and $f \neq 0$ in X, $vf = \lambda f$, then $f(x) = \lambda f'(x)$. So $\lambda \neq 0$ and $lnf(x) = \frac{x}{\lambda} + c$ for some $c \in \mathcal{R}$. Hence $f(x) = f(0)e^{\frac{x}{\lambda}} = 0e^{\frac{x}{\lambda}} = 0$, a contradiction. v has then no eigenvalue.