An unbounded operator on a Hilbert space H annihilating an orthonormal basis of H.

****

Let (e_n) be the standard orthonormal basis for the Hilbert space $H = l^2$. Extend (e_n) to a Hamel basis β for l^2. Choose $f \in \beta$ distinct to the e_n and define then the linear mapping $T : H \longrightarrow H$ by

$$T(g) = \begin{cases} 1 & g = f \\ 0 & g \in \beta \setminus \{f\} \end{cases}$$

Then $T(e_n) = 0$ and T is unbounded (otherwise, $1 = T(f) = \sum_{n=1}^{\infty} < f, e_n > Te_n = 0$).