An operator U on a Hilbert space, other than I, such that $sp(U) = \{1\}$ and $\| U \|= 1$.

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Suppose that $H = L^2(0,1)$ with respect to the Lebesgue measure and $(Tf)(x) = \int_0^x f(t)dt$. It follows from BA15.DVI, $sp(T) = 0$, so that $sp(I + T) = \{1\}$. Hence $U = (I+T)^{-1} \neq I$ is well-defined, moreover $sp(U) = \{\lambda^{-1}; \lambda \in sp(I+T)\} = \{1\}$. Therefore

$$1 = r(U) \leq \| U \| .$$

But $\| U \| \leq 1$, since

$$\| U^{-1}\xi \|^2 = \| (I+T)\xi \|^2 = \| f \|^2 + < (T+T^*)\xi, \xi > + \| T\xi \|^2 \geq \| f \|^2 .$$

(Note that $T + T^*$ is a projection onto the space of constant functions, since $(T^*f)(t) = \int_t^1 f(t)dt$.)
Thus $\| U \| = 1$.