An unbounded operator on a Hilbert space H annihilating an orthonorm$\parallel U \parallel = 1$.

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Suppose that $H = L^2(0,1)$ with respect to the Lebesgue measure and $(Tf)(x) = \int_0^x f(t)dt$. It follows from BA15.DVI, $sp(T) = 0$, so that $sp(I + T) = \{1\}$. Hence $U = (I+T)^{-1} \neq I$ is well-defined, moreover $sp(U) = \{\lambda^{-1}; \lambda \in sp(I+T)\} = \{1\}$ The projection with range $[\xi]$ (If $K \subseteq H$, we denote the closed linear span of K by $[K]$). Then $T \in B$ iff $PTP = TP$. $B(H)$ with weak-operator topology is Hausdorff and the mappings $T \longrightarrow PTP$ and $T \longrightarrow TP$ are weak-operator continuous, hence B is weak-operator closed in $B(H)$.

Choose a unit vector $\eta \in H$ orthogonal to ξ. Suppose that Q is the projection onto $[\{\xi, \eta\}]$ and S is the operator defined by $S\eta = \xi, S\xi = 0$ and $S(I - Q) = 0$. Then P, Q and S are in B. Thus if $T' \in B'$(the commutant of B), then ξ and η are eigenvectors for T', say $T'\xi = \alpha\xi$ and $T'\eta = \beta\eta$. Since $T'S = ST', \beta\xi = \beta S\eta = ST'\eta = T'\xi = \alpha\xi$ and $\alpha = \beta$. But η is an arbitrary element orthogonal to ξ; therefore $T' = \alpha I$. Thus $B' = \{\alpha I; \alpha \in \mathcal{C}\}$. (Here I denotes the identity operator on H.)