A unitary operator U acting on a Hilbert space whose spectrum is $C = \{z \in \mathcal{C}; |z| = 1\}$.

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If H is a separable infinite dimensional Hilbert space with an orthogonal basis $(\xi_n)_{n\in\mathcal{Z}}$, we define $U\xi_n = \xi_{n+1}$. Then U is isometric and surjective, so it is a unitary operator. By Lemma 3.2.13 of [K&R1], $sp(U) \subseteq C$. If $\lambda \in C$ and $x_n = (2n+1)^{\frac{-1}{2}} \sum_{k=-n}^{n} \lambda^{-k}\xi_k$, then $\| \xi_n \| = 1$ and $\| (U - \lambda I)x_n \| = (2n + 1)^{\frac{-1}{2}} \| \sum_{k=-n}^{n} \lambda^{-k}\xi_{k+1} - \sum_{k=-n}^{n} \lambda^{-(k-1)}\xi_k \| = (2n+1)^{\frac{-1}{2}} \| \lambda^{-n}\xi_{n+1} - \lambda^{n+1}\xi_{-n} \| = 2^{\frac{1}{2}}(2n+1)^{\frac{-1}{2}} \to 0$.

Therefore by the same lemma, $\lambda \in sp(U)$. Thus $sp(U) = C$.

Ref.

[K&R1] R.V. Kadison and J.R. Ringrose, Fundamental of the theory of Operator Algebras (I), Acad. Press, 1983.