

Two selfadjoint operators T and S on a Hilbert space such that $sp(ST)$ is not a subset of \mathcal{R} .

Consider

$$S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

belonging to $B(\mathcal{C}^2)$. Then S and T are Hermetian, but $sp(ST) = \{i, -i\}$ which is not a subset of \mathcal{R} .

