Two Hermetian operators T and S on a Hilbert space such that $S \geq 0$ and $-S \leq T \leq S$ but not $|T| \leq S$.

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Let

$$S = \begin{pmatrix} 3 & 2 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & \sqrt{2} \\ 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \sqrt{2} & \sqrt{2} \end{pmatrix} \geq 0, T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(S and T belong to $B(\mathcal{C}^2)$.)

Then

$$S - T = \begin{pmatrix} \sqrt{2} & 0 \\ \sqrt{2} & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & 1 \end{pmatrix} \geq 0, S + T = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix} \geq 0$$

and so $-S \leq T \leq S$, but $S - |T| = \begin{pmatrix} 2 & 2 \\ 2 & 1 \end{pmatrix}$ and $< (S - |T|)\xi, \xi >= -1$ where $\xi = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, hence S dosen't majorize $|T|$.