

A selfadjoint operator $T \neq 0$ on a Hilbert space such that T is neither positive nor negative.

Consider $T = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ belonging to $B(\mathcal{C}^2)$, $\xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\eta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.
Then $\langle T\xi, \xi \rangle = -1$ and $\langle T\eta, \eta \rangle = 1$. Hence the selfadjoint operator T is neither positive nor negative.