A unital commutative Banach algebra with a maximal ideal $M$ of codimension 1 and a Banach $A$-module $X$ such that $H^2(A, X) = 0$ but $H^2(M, X) \neq 0$.

Let $A = \mathbb{C}^2$ with the product $(a, b)(a', b') = (aa', ab' + a'b)$. $M = \{0\} \oplus \mathbb{C}$, being the kernel of the character $\phi : A \rightarrow \mathbb{C}$ defined by $\phi(z, w) = z$, is a maximal ideal of codimension 1. Regard $X = \mathbb{C}$ as an annihilator $A$-module. By [B&D&L, Proposition 2.2],

$H^2(A, X) = \{0\}$. If $\mu((0, w_1), (0, w_2)) = w_1w_2$, then $\mu \in Z^2(M, X)$, but $\mu \notin N^2(M, X)$ (otherwise $w_1w_2 = \mu((0, w_1), (0, w_2)) = (\delta^1\lambda)((0, w_1), (0, w_2)) = (0, w_1)\lambda((0, w_2)) - \lambda((0, w_1), (0, w_2)) + \lambda(0, w_1).(0, w_2) = 0 - \lambda(0) + 0 = 0$, for all $w_1, w_2 \in \mathbb{C}$, a contradiction).

Thus $H^2(M, X) \neq 0$.

Ref.