A unital commutative Banach algebra with a maximal ideal M of codimension 1 and a Banach A-module X such that $H^2(A,X)=0$ but $H^2(M,X)\neq 0$.

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Let $A=\mathcal{C}^2$ with the product $(a,b)(a',b')=(aa',ab'+a'b)$. $M=\{0\}\oplus\mathcal{C}$, being the kernel of the character $\phi : A \longrightarrow \mathcal{C}$ defined by $\phi(z,w)=z$, is a maximal ideal of codimension 1. Regard $X=\mathcal{C}$ as an annihilator A-module. By [B&D&L, Proposition 2.2],

$H^2(A,X)=\{0\}$. If $\mu((0,w_1),(0,w_2))=w_1w_2$, then $\mu\in Z^2(M,X)$, but $\mu\notin N^2(M,X)$ (otherwise $w_1w_2=\mu((0,w_1),(0,w_2))=(\delta^1\lambda)((0,w_1),(0,w_2))=(0,w_1).\lambda((0,w_2))-\lambda((0,w_1).(0,w_2))+\lambda(0,w_1).(0,w_2)=0-\lambda(0)+0=0$, for all $w_1,w_2\in\mathcal{C}$, a contradiction).

Thus $H^2(M,X)\neq 0$.

Ref.

[B&D&L] W.G. Bode, H.G. Dales and Z. Lykova, Algebraic and strany splittings of extensions of Banach algebras, mem. Amer. Math. Soc. 137 (1999).