A unital commutative Banach algebra with a maximal ideal M of codimension 1 and a Banach A-module X such that $H^2(A,X) = 0$ but $H^2(M,X) \neq 0$.

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Let $A = \mathcal{C}^2$ with the product $(a,b)(a',b') = (aa', ab' + a'b)$. $M = \{0\} \oplus \mathcal{C}$, being the kernel of the character $\phi : A \longrightarrow \mathcal{C}$ defined by $\phi(z,w) = z$, is a maximal ideal of codimension 1. Regard $X = \mathcal{C}$ as an annihilator A-module. By [B&D&L, Proposition 2.2],

$H^2(A,X) = \{0\}$. If $\mu((0,w_1),(0,w_2)) = w_1w_2$, then $\mu \in Z^2(M,X)$, but $\mu \notin N^2(M,X)$ (otherwise $w_1w_2 = \mu((0,w_1),(0,w_2)) = (\delta^1\lambda)((0,w_1),(0,w_2)) = (0,w_1).\lambda((0,w_2)) - \lambda((0,w_1).(0,w_2)) + \lambda(0,w_1).(0,w_2) = 0 - \lambda(0) + 0 = 0$, for all $w_1, w_2 \in \mathcal{C}$, a contradiction).

Thus $H^2(M,X) \neq 0$.

Ref.

[B&D&L] W.G. Bode, H.G. Dales and Z. Lykova, Algebraic and strany splittings of extensions of Banach algebras, mem. Amer. Math. Soc. 137 (1999).