A non-split short complex of Banach spaces whose dual splits.

\[ 0 \longrightarrow c_0 \xrightarrow{i} \ell^\infty \xrightarrow{\pi} \frac{\ell^\infty}{c_0} \longrightarrow 0 \] is a short exact complex of Banach spaces which doesn’t split since \( c_0 \) is not complemented in \( \ell^\infty \).

Its dual complex \[ 0 \longrightarrow (\frac{\ell^\infty}{c_0})^\# \xrightarrow{\pi^\#} (\ell^\infty)^\# = (l^1)^{##} \xrightarrow{i^\#} c_0^\# = l^1 \longrightarrow 0 \] splits. Notice that the later complex is exact and the canonical embedding \( l^1 \longrightarrow (l^1)^{##} \) is a right inverse to \( i^\# \).