A non-closable unbounded $*$-derivation.

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Let X be the Cantor set of $[0,1]$. It is well-known that X is a perfect compact subset of $[0,1]$. By Tietze's theorem, $C(X) = \{f|_X; f \in C([0,1])\}$. Define δ on $D(\delta) = \{f|_X; f \in C^1([0,1])\}$ by $\delta(f|_X) = f'|_X$. δ is a well-defined derivation (if $f|_X = 0$, then for each $x_0 \in X$ there exists a sequence $\{x_n\}$ in $X - \{x_0\}$ converging to x_0. So $f'(x_0) = \lim_n \frac{f(x_n) - f(x_0)}{x_n - x_0} = 0$, therefore $f'|_X = 0$).

But δ is not identically zero so by [Sak2, Proposition 3.2.1] δ cannot be extended to a closed derivation in $C(X)$. So that δ isn't closable.

This example is due to O.Bratteli and D.W. Robinson ([cf. Sak2, P.59]).

Ref.

[Sak2] S. Sakai, Operator algebras in dynamical systems, Cambridge Univ. Press, 1991.