Linear Optimization

Homework 3

Deadline 15 Apr 2012

1. Exercises 8.2, 8.3, 8.9, 8.12, 8.22, 8.23, 8.24, 8.26 and 8.30 in [3] pages 399-406.

2. Consider a feasible solution x to a standard form problem, and let $Z = \{i : x_i = 0\}$. Show that x is an optimal solution if and only if the linear programming problem $\min c^T d$ subject to $Ad = 0,\ d_i \geq 0, i \in Z$, has an optimal cost of zero. (In this sense, deciding optimality is equivalent to solving a new linear programming problem.) [1].

3. Let A be a symmetric square matrix. Consider the linear programming problem
$$\begin{cases} \min c^T x \\ \text{s.t.} \\ Ax \geq c \\ x \geq 0. \end{cases}$$
Prove that if x^* satisfies $Ax^* = c$ and $x^* \geq 0$, then x^* is an optimal solution [1].

4. *(Ph.D. Comprehensive Exam, September 21, 1991, at Stanford).* Consider the two linear programs (i) min $0^T x$ subject to $Ax \geq 0, x \geq 0$, and (ii) max $0^T y$ subject to $A^T y \leq 0,\ y \geq 0$. (a) Prove that either program is the dual of the other. (b) Prove that either there exists an $x \geq 0$ such that $Ax > 0$ or there exists a nontrivial solution to the dual linear program [2].

References

[1] D. Bertsimas and J.N. Tsitsiklis, *Introduction to linear optimization*, thena Secientific Sereis, 1997.

[2] G. B. Dantzig and M. N. Thapa, *Linear programming*, Springer, 2003.

[3] P.E. Gill, W. Murray, and M. H. Wright, *Numerical linear algebra and optimization*, vol. 1, Perseus Books (Sd), 1990.