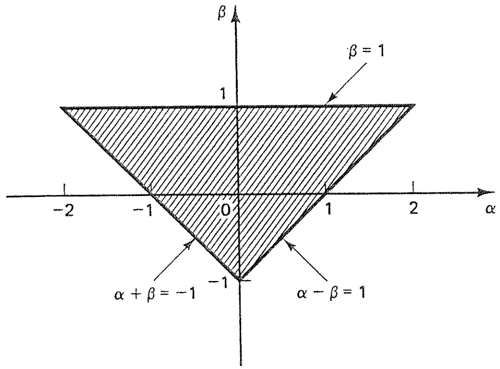

Figure 2–7 Region in the $\alpha\beta$ plane in which the solution series of Equation (2–32), subjected to initial conditions, is finite.

Find the conditions on α and β for which the solution series $x(k)$ for $k = 0, 1, 2, \ldots$, subjected to initial conditions, is finite.

Solution Let us define

$$\alpha = a + b, \qquad \beta = ab$$

Then, referring to Example 2–19, the solution $x(k)$ for $k = 0, 1, 2, \ldots$ can be given by

$$x(k) = \begin{cases} \dfrac{bx(0) + x(1)}{b - a}(-a)^k + \dfrac{ax(0) + x(1)}{a - b}(-b)^k, & a \neq b \\ x(0)(-a)^k + [ax(0) + x(1)]k(-a)^{k-1}, & a = b \end{cases}$$

The solution series $x(k)$ for $k = 0, 1, 2, \ldots$, subjected to initial conditions $x(0)$ and $x(1)$, is finite if the absolute values of a and b are less than unity. Thus, on the $\alpha\beta$ plane, three critical points can be located:

$$\alpha = 2, \qquad \beta = 1$$

$$\alpha = -2, \qquad \beta = 1$$

$$\alpha = 0, \qquad \beta = -1$$

The interior of the region bounded by lines connecting these points satisfies the condition $|a| < 1, |b| < 1$. The boundary lines can be given by $\beta = 1$, $\alpha - \beta = 1$, and $\alpha + \beta = -1$. See Figure 2–7. If point (α, β) lies inside the shaded triangular region, then the solution series $x(k)$ for $k = 0, 1, 2, \ldots$, subjected to initial conditions $x(0)$ and $x(1)$, is finite.

PROBLEMS

Problem B–2–1

Obtain the z transform of

$$x(t) = \frac{1}{a}(1 - e^{-at})$$

where a is a constant.

Problem B–2–2

Obtain the z transform of k^3.

Problem B–2–3

Obtain the z transform of $t^2 e^{-at}$.

Problem B–2–4

Obtain the z transform of the following $x(k)$:

$$x(k) = 9k(2^{k-1}) - 2^k + 3, \qquad k = 0, 1, 2, \ldots$$

Assume that $x(k) = 0$ for $k < 0$.

Problem B–2–5

Find the z transform of

$$x(k) = \sum_{h=0}^{k} a^h$$

where a is a constant.

Problem B–2–6

Show that

$$\mathcal{Z}[k(k-1)a^{k-2}] = \frac{(2!)z}{(z-a)^3}$$

$$\mathcal{Z}[k(k-1)\cdots(k-h+1)a^{k-h}] = \frac{(h!)z}{(z-a)^{h+1}}$$

Problem B–2–7

Obtain the z transform of the curve $x(t)$ shown in Figure 2–8.

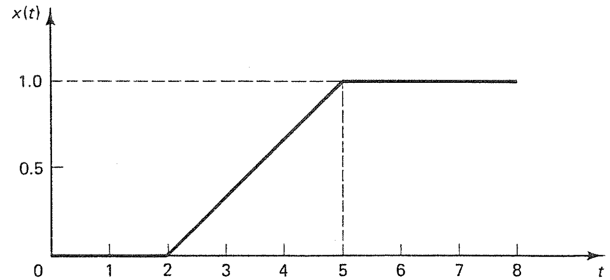

Figure 2–8 Curve $x(t)$.

Problem B–2–8

Obtain the inverse z transform of

$$X(z) = \frac{1 + 2z + 3z^2 + 4z^3 + 5z^4}{z^4}$$

Problem B–2–9

Find the inverse z transform of

$$X(z) = \frac{z^{-1}(0.5 - z^{-1})}{(1 - 0.5z^{-1})(1 - 0.8z^{-1})^2}$$

Use (1) the partial-fraction-expansion method and (2) the MATLAB method. Write a MATLAB program for finding $x(k)$, the inverse z transform of $X(z)$.

Problem B–2–10

Given the z transform

$$X(z) = \frac{z^{-1}}{(1 - z^{-1})(1 + 1.3z^{-1} + 0.4z^{-2})}$$

determine the initial and final values of $x(k)$. Also find $x(k)$, the inverse z transform of $X(z)$, in a closed form.

Problem B–2–11

Obtain the inverse z transform of

$$X(z) = \frac{1 + z^{-1} - z^{-2}}{1 - z^{-1}}$$

Use (1) the inversion integral method and (2) the MATLAB method.

Problem B–2–12

Obtain the inverse z transform of

$$X(z) = \frac{z^{-3}}{(1 - z^{-1})(1 - 0.2z^{-1})}$$

in a closed form.

Problem B–2–13

By using the inversion integral method, obtain the inverse z transform of

$$X(z) = \frac{1 + 6z^{-2} + z^{-3}}{(1 - z^{-1})(1 - 0.2z^{-1})}$$

Problem B–2–14

Find the inverse z transform of

$$X(z) = \frac{z^{-1}(1 - z^{-2})}{(1 + z^{-2})^2}$$

Use (1) the direct division method and (2) the MATLAB method.

Problem B–2–15

Obtain the inverse z transform of

$$X(z) = \frac{0.368z^2 + 0.478z + 0.154}{(z - 1)z^2}$$

by use of the inversion integral method.

Problem B–2–16

Find the solution of the following difference equation:

$$x(k + 2) - 1.3x(k + 1) + 0.4x(k) = u(k)$$

where $x(0) = x(1) = 0$ and $x(k) = 0$ for $k < 0$. For the input function $u(k)$, consider the following two cases:

$$u(k) = \begin{cases} 1, & k = 0, 1, 2, \ldots \\ 0, & k < 0 \end{cases}$$

and

$$u(0) = 1$$
$$u(k) = 0, \qquad k \neq 0$$

Solve this problem both analytically and computationally with MATLAB.

Problem B–2–17

Solve the following difference equation:

$$x(k + 2) - x(k + 1) + 0.25x(k) = u(k + 2)$$

where $x(0) = 1$ and $x(1) = 2$. The input function $u(k)$ is given by

$$u(k) = 1, \qquad k = 0, 1, 2, \ldots$$

Solve this problem both analytically and computationally with MATLAB.

Problem B–2–18

Consider the difference equation:

$$x(k + 2) - 1.3679x(k + 1) + 0.3679x(k) = 0.3679u(k + 1) + 0.2642u(k)$$

where $x(k) = 0$ for $k \leq 0$. The input $u(k)$ is given by

$$u(k) = 0, \qquad k < 0$$
$$u(0) = 1.5820$$
$$u(1) = -0.5820$$
$$u(k) = 0, \qquad k = 2, 3, 4, \ldots$$

Determine the output $x(k)$. Solve this problem both analytically and computationally with MATLAB.