PROBLEMS

Problem B–3–1

Show that the circuit shown in Figure 3–64 acts as a zero-order hold.

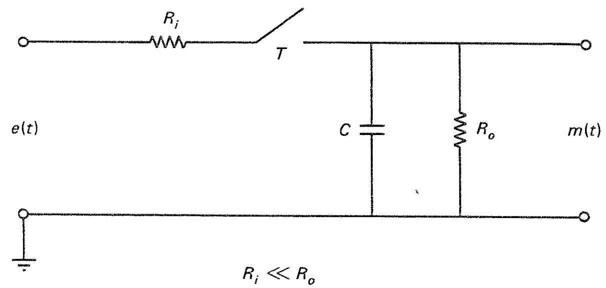

Figure 3–64 Circuit approximating a zero-order hold.

Problem B–3–2

Consider the circuit shown in Figure 3–65. Derive a difference equation describing the system dynamics when the input voltage applied is piecewise constant, or

$$e(t) = e(kT), \qquad kT \le t < (k+1)T$$

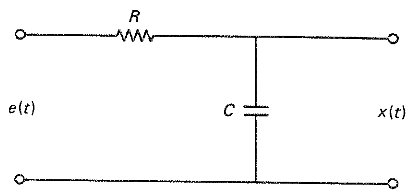

Figure 3–65 *RC* circuit.

(Derive first a differential equation and then discretize it to obtain a difference equation.)

Problem B–3–3

Consider the impulse sampler and first-order hold shown in Figure 3–66. Derive the transfer function of the first-order hold, assuming a unit-ramp function as the input $x(t)$ to the sampler.

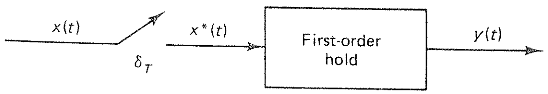

Figure 3–66 Impulse sampler and first-order hold.

Problem B–3–4

Consider a transfer function system

$$X(s) = \frac{s+3}{(s+1)(s+2)}$$

Obtain the pulse transfer function by two different methods.

Problem B–3–5

Obtain the z transform of

$$X(s) = \frac{K}{(s+a)(s+b)}$$

Use the residue method and the method based on the impulse response function.

Problem B–3–6

Obtain the z transform of

$$X(s) = \frac{1-e^{-Ts}}{s}\frac{1}{(s+a)^2}$$

Problem B–3–7

Consider the difference equation system

$$y(k+1) + 0.5y(k) = x(k)$$

where $y(0) = 0$. Obtain the response $y(k)$ when the input $x(k)$ is a unit-step sequence. Also, obtain the MATLAB solution.

Problem B–3–8

Consider the difference equation system

$$y(k+2) + y(k) = x(k)$$

where $y(k) = 0$ for $k < 0$. Obtain the response $y(k)$ when the input $x(k)$ is a unit-step sequence. Also, obtain the MATLAB solution.

Problem B–3–9

Obtain the weighting sequence $g(k)$ of the system described by the difference equation

$$y(k) - ay(k-1) = x(k), \qquad -1 < a < 1$$

If two systems described by this last equation are connected in series, what is the weighting sequence of the resulting system?

Problem B–3–10

Consider the system described by

$$y(k) - y(k-1) + 0.24y(k-2) = x(k) + x(k-1)$$

where $x(k)$ is the input and $y(k)$ is the output of the system.

Determine the weighting sequence of the system. Assuming that $y(k) = 0$ for $k < 0$, determine the response $y(k)$ when the input $x(k)$ is a unit-step sequence. Also, obtain the MATLAB solution.

Problem B–3–11

Consider the system

$$G(z) = \frac{1 - 0.5z^{-1}}{(1 - 0.3z^{-1})(1 + 0.7z^{-1})}$$

Obtain the response of this system to a unit-step sequence input. Also, obtain the MATLAB solution.

Problem B–3–12

Obtain the response $y(kT)$ of the following system:

$$\frac{Y(s)}{X^*(s)} = \frac{1}{(s + 1)(s + 2)}$$

where $x(t)$ is the unit-step function and $x^*(t)$ is its impulse-sampled version. Assume that the sampling period T is 0.1 sec.

Problem B–3–13

Consider the system defined by

$$\frac{Y(z)}{U(z)} = H(z) = \frac{0.5z^3 + 0.4127z^2 + 0.1747z - 0.0874}{z^3}$$

Using the convolution equation

$$y(k) = \sum_{j=0}^{k} h(k - j)u(j)$$

obtain the response $y(k)$ to a unit-step sequence input $u(k)$.

Problem B–3–14

Assume that a sampled signal $X^*(s)$ is applied to a system $G(s)$. Assume also that the output of $G(s)$ is $Y(s)$ and $y(0+) = 0$.

$$Y(s) = G(s)X^*(s)$$

Using the relationship

$$Y^*(s) = \frac{1}{T}\sum_{k=-\infty}^{\infty} Y(s + j\omega_s k)$$

show that

$$Y^*(s) = G^*(s)X^*(s)$$

Problem B–3–15

Obtain the closed-loop pulse transfer function of the system shown in Figure 3–67.

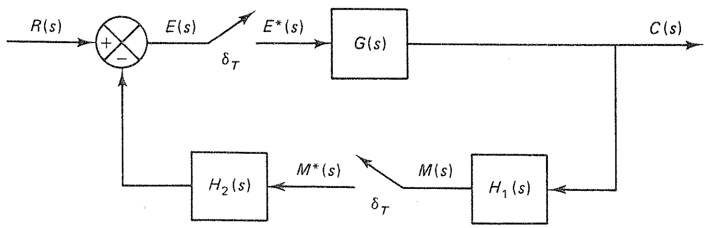

Figure 3–67 Discrete-time control system.

Problem B–3–16

Obtain the closed-loop pulse transfer function of the system shown in Figure 3–68.

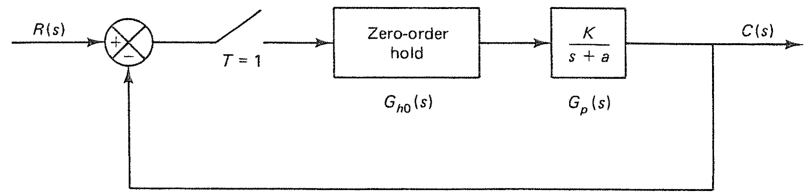

Figure 3–68 Discrete-time control system.

Problem B–3–17

Consider the discrete-time control system shown in Figure 3–69. Obtain the discrete-time output $C(z)$ and the continuous-time output $C(s)$ in terms of the input and the transfer functions of the blocks.

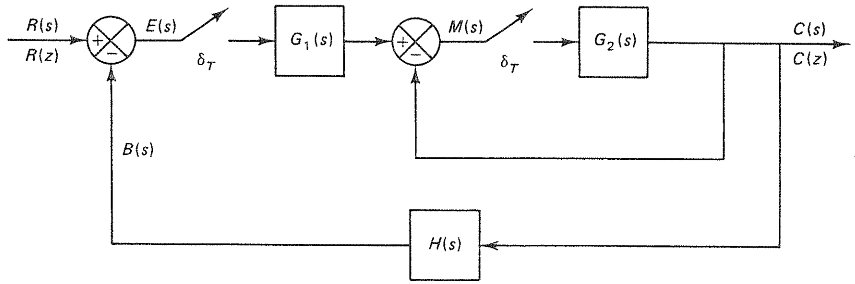

Figure 3–69 Discrete-time control system.

Problem B–3–18

Consider the discrete-time control system shown in Figure 3–70. Obtain the output sequence $c(kT)$ of the system when it is subjected to a unit-step input. Assume that the sampling period T is 1 sec. Also, obtain the continuous-time output $c(t)$.

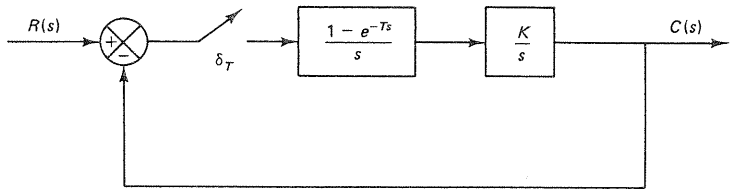

Figure 3–70 Discrete-time control system.

Problem B–3–19

Obtain in a closed form the response sequence $c(kT)$ of the system shown in Figure 3–71 when it is subjected to a Kronecker delta input $r(k)$. Assume that the sampling period T is 1 sec.

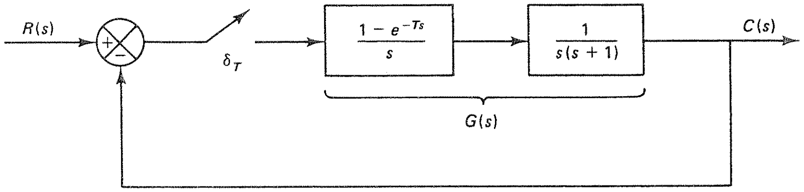

Figure 3–71 Discrete-time control system.

Problem B–3–20

Consider the system shown in Figure 3–72. Assuming that the sampling period T is 0.2 sec and the gain constant K is unity, determine the response $c(kT)$ for $k = 0, 1, 2, 3,$ and 4 when the input $r(t)$ is a unit-step function. Also, determine the final value $c(\infty)$.

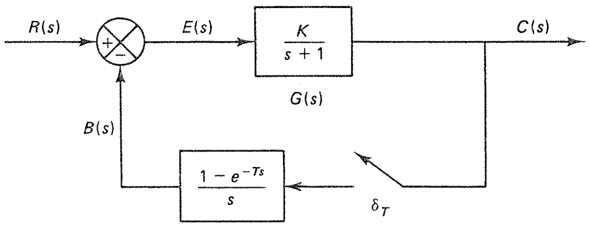

Figure 3–72 Discrete-time control system.

Problem B–3–21

Obtain the closed-loop pulse transfer function $C(z)/R(z)$ of the digital control system shown in Figure 3–30. Assume that the pulse transfer function of the plant is $G(z)$. (Note that the system shown in Figure 3–30 is a velocity-form PID control of the plant.)

Problem B–3–22

Assume that a digital filter is given by the following difference equation:

$$y(k) + a_1 y(k-1) + a_2 y(k-2) = b_1 x(k) + b_2 x(k-1)$$

Draw block diagrams for the filter using (1) direct programming, (2) standard programming, and (3) ladder programming.

Problem B–3–23

Consider the digital filter defined by

$$G(z) = \frac{2 + 2.2z^{-1} + 0.2z^{-2}}{1 + 0.4z^{-1} - 0.12z^{-2}}$$

Realize this digital filter in the series scheme, the parallel scheme, and the ladder scheme.

Problem B–3–24

Referring to the approximate differentiator shown in Figure 3–63, draw a graph of the output $v(k)$ versus k when the input $x(k)$ is a unit-step sequence.

Problem B–3–25

Consider the system shown in Figure 3–73. Show that the pulse transfer function $Y(z)/X(z)$ is given by

$$\frac{Y(z)}{X(z)} = T\left(\frac{1}{1 - z^{-1}}\right)$$

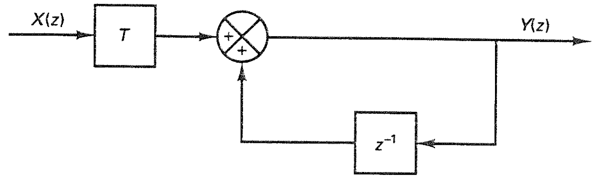

Figure 3–73 Digital integrator without delay.

Assuming that $y(kT) = 0$ for $k < 0$, show that

$$y(kT) = T[x(0) + x(T) + \cdots + x(kT)]$$

Thus, the output $y(kT)$ approximates the area made by the input. Hence, the system acts as an integrator. Because $y(0) = Tx(0)$, the output appears as soon as $x(0)$ enters the system. This integrator is commonly called a digital integrator without delay.

Draw a graph of the output $y(kT)$ when the input $x(kT)$ is a unit-step sequence.

Problem B–3–26

Consider the system shown in Figure 3–74. Show that the pulse transfer function $Y(z)/X(z)$ is given by

$$\frac{Y(z)}{X(z)} = T\left(\frac{z^{-1}}{1 - z^{-1}}\right)$$

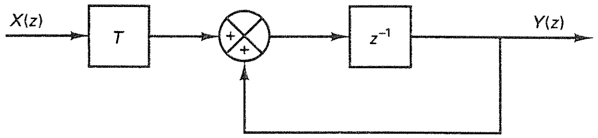

Figure 3–74 Digital integrator with delay.

Assuming that $y(kT) = 0$ for $k < 0$, show that

$$y(kT) = T[x(0) + x(T) + \cdots + x((k-1)T)]$$

The output $y(kT)$ approximates the area made by the input. Since $y(0) = 0$ and $y(T) = Tx(0)$, the output starts to appear at $t = T$. This integrator is called a digital integrator with delay.

Draw a graph of the output $y(kT)$ when the input $x(kT)$ is a unit-step sequence.

Problem B–3–27

Consider the system shown in Figure 3–75. Show that the pulse transfer function $Y(z)/X(z)$ is given by

$$\frac{Y(z)}{X(z)} = \frac{T}{2}\left(\frac{1}{1 - z^{-1}} + \frac{z^{-1}}{1 - z^{-1}}\right)$$

This system is a combination of the digital integrators without delay and with delay, as presented in Problems B–3–25 and B–3–26, respectively.

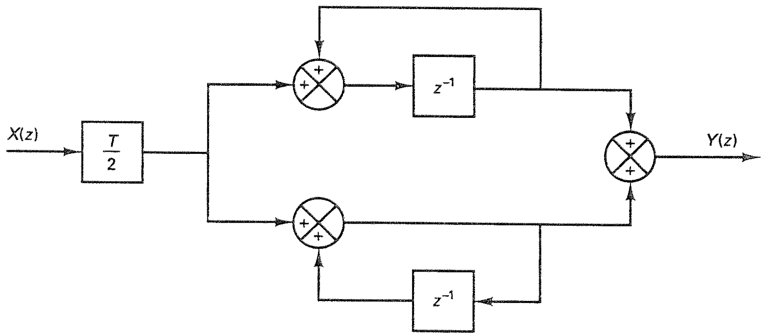

Figure 3–75 Digital bilinear integrator.

Assuming that $y(kT) = 0$ for $k < 0$, obtain $y(kT)$ in terms of $x(0), x(T), \ldots, x(kT)$. This integrator is called a digital bilinear integrator. Draw a graph of the output $y(kT)$ when the input $x(kT)$ is a unit-step sequence.

4

Design of Discrete-Time Control Systems by Conventional Methods

4–1 INTRODUCTION

In this chapter we first present mapping from the s plane to the z plane and then discuss stability of closed-loop control systems in the z plane. Next we treat three different design methods for single-input–single-output discrete-time or digital control systems. The first method is based on the root-locus technique using pole–zero configurations in the z plane. The second method is based on the frequency-response method in the w plane. The third method is an analytical method in which we attempt to obtain a desired behavior of the closed-loop system by manipulating the pulse transfer function of the digital controller.

Design techniques for continuous-time control systems based on conventional transform methods (the root-locus and frequency-response methods) have become well established since the 1950s. Conventional transform methods are especially useful for designing industrial control systems. In fact, in the past, many industrial digital control systems were successfully designed on the basis of conventional transform methods. Both familiarity with the root-locus and frequency-response techniques and experiences gained in the design of analog controllers are immensely valuable in designing discrete-time control systems.

Outline of the Chapter. Section 4–1 has presented introductory material. Section 4–2 treats mapping from the s plane to the z plane. Section 4–3 discusses the Jury stability criterion for closed-loop control systems in the z plane. Section 4–4 summarizes transient and steady-state response characteristics of discrete-time control systems. The design technique based on the root-locus method is presented in Section 4–5. Section 4–6 first reviews the frequency-response method and then presents frequency-response techniques using the w transformation for designing discrete-time control systems. Section 4–7 treats an analytical design method.