kinematics

Meghdadi Fall 2016

How do we describe motion?

- ✓Position
- ✓Displacement
- ✓Traveled distance
- ✓Speed
- ✓Velocity
- ✓Acceleration

Position

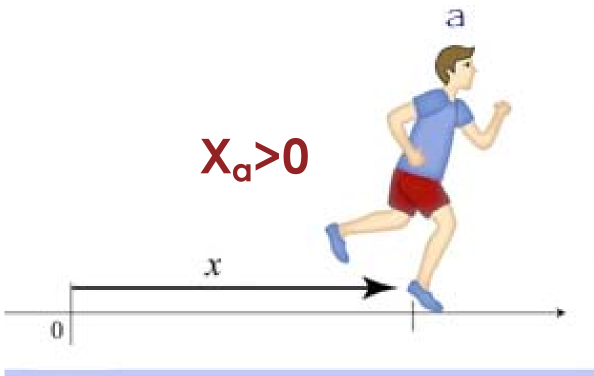
- The **position** of a point mass would be defined vs an **origin**.

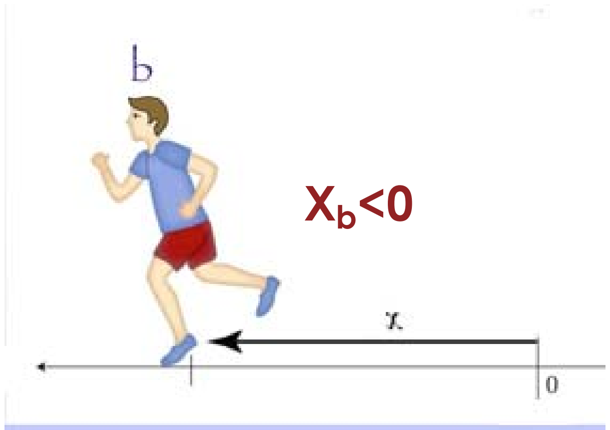
- The positive direction can be chosen as desired.

- The position is a vector quantity. It means that it has both a direction and a magnitude.

Positive direction

$X_a>0$





$X_b<0$

Displacement vs Traveled distance

$\Delta X_1 = \Delta X_2 = \Delta X_3 = X_B - X_A$

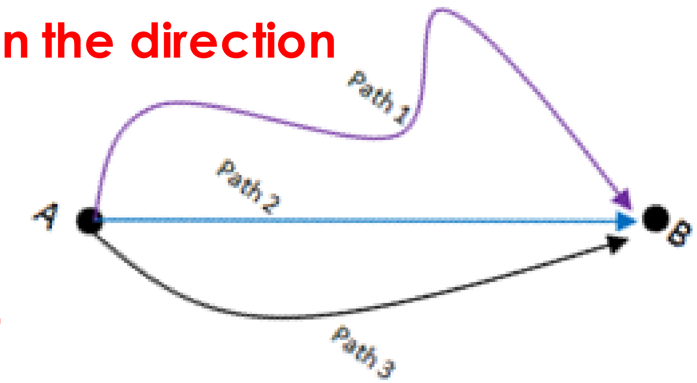
$\Delta X_O = \Delta X_{O'}$

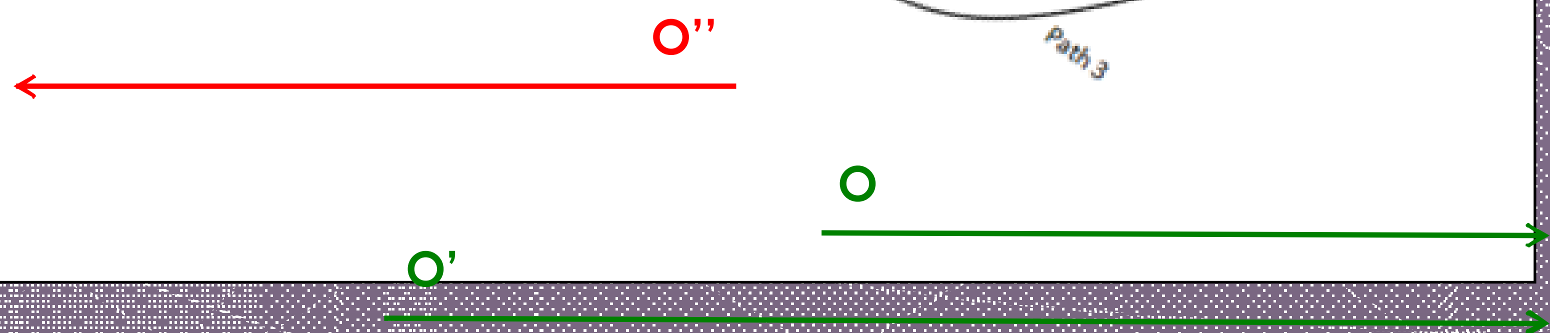
$\Delta X_O = -\Delta X_{O''}$

Displacement is independent of path.

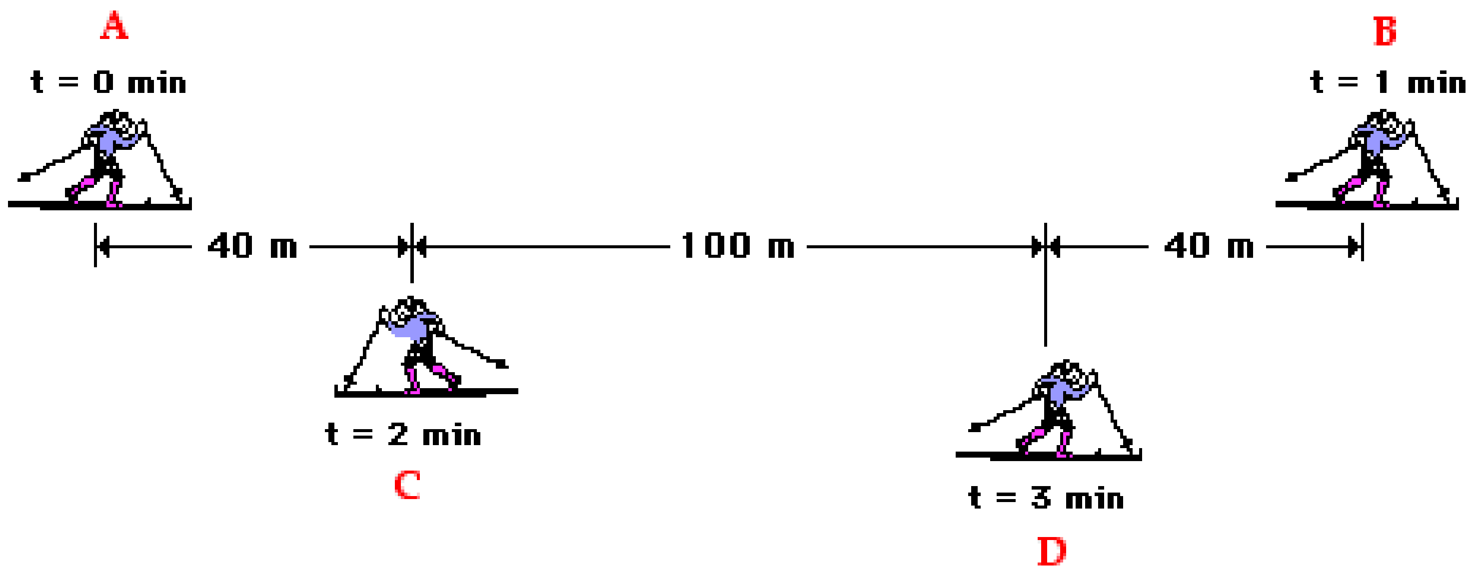
Displacement is independent of origin.

Displacement is <u>dependent</u> on the direction





Example



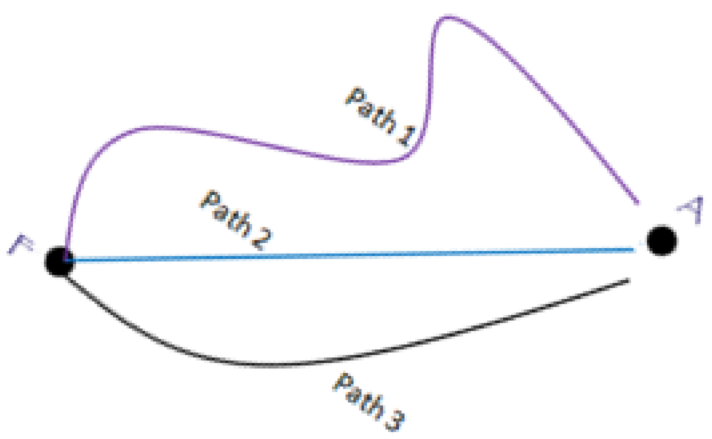
$$\Delta X_{AD} = X_D - X_A = 140 - 0 = 140 \text{ m}$$

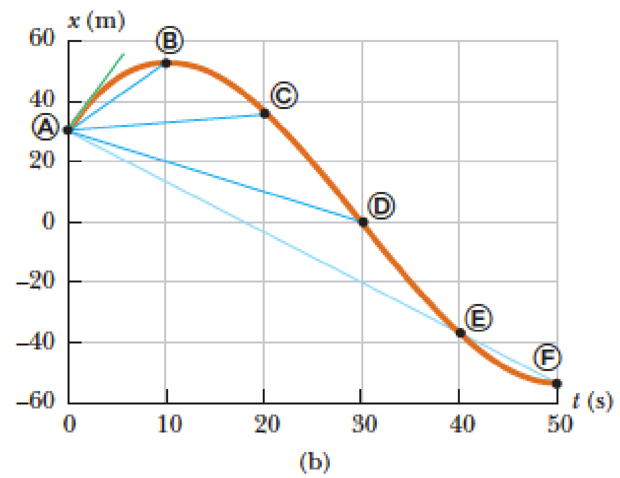
$$\text{Traveled Distance}_{AD} = |\Delta X_{AB}| + |\Delta X_{BC}| + |\Delta X_{CD}|$$

$$= 180 + 140 + 100 = 420 \text{ m}$$

Traveled Distance is an scalar quantity.

Average velocity vs Average Speed





Average $\text{Velocity}_{AF} = V_{AF} = (X_F - X_A) / (t_F - t_A)$

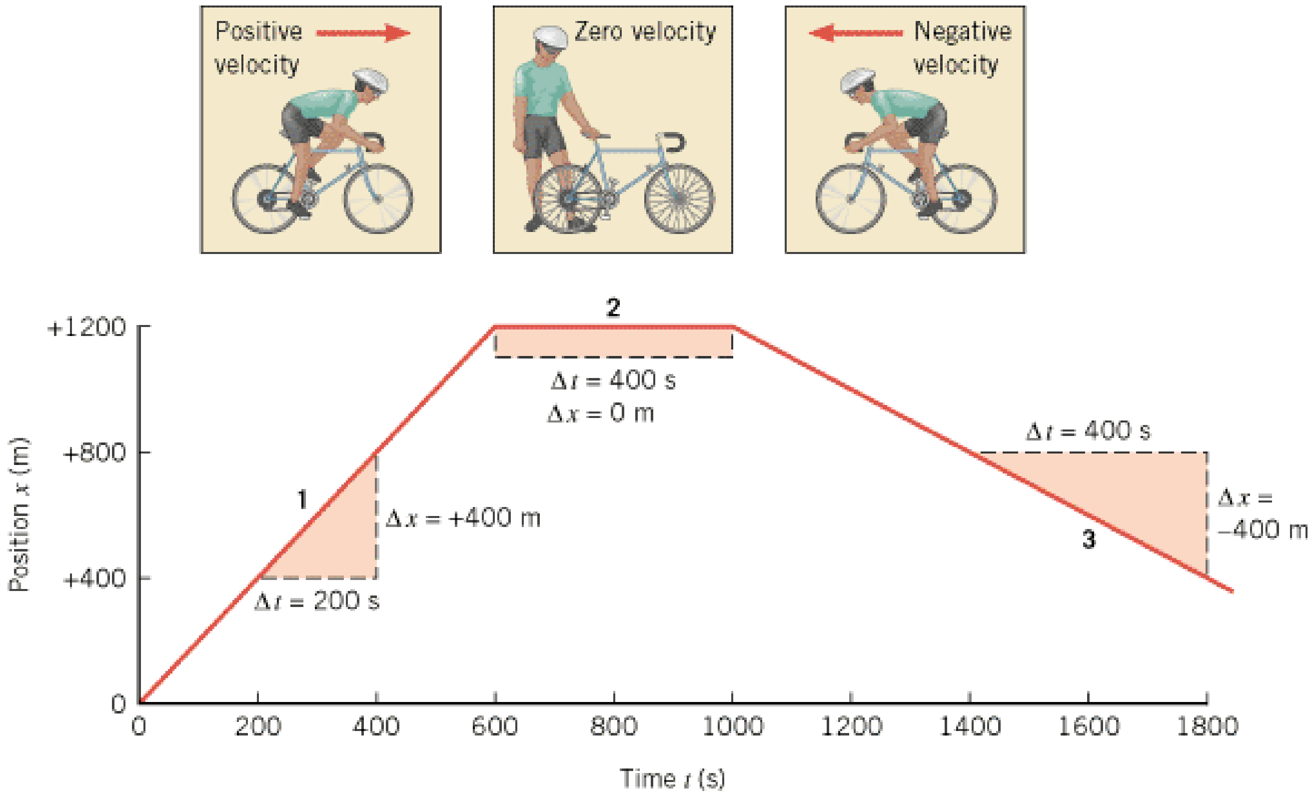
$= (-50-30) / (50-0) = -1.6$ m/s

Average Speed_{AF} (path1)= (length of Path1)/$(t_F - t_A)$

(path2)= (length of Path2)/$(t_F - t_A)$

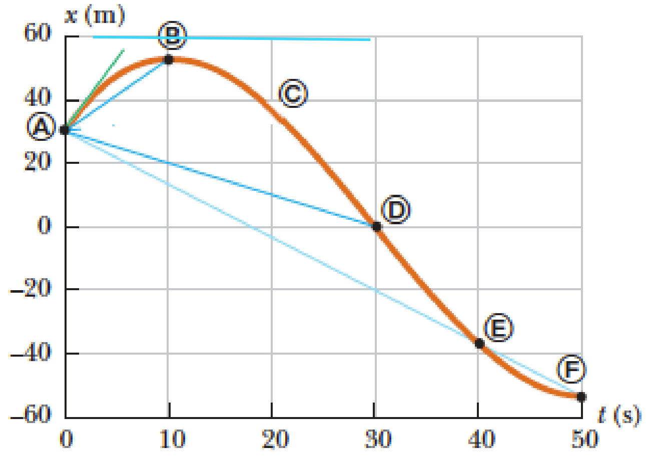
(path3)= (length of Path3)/$(t_F - t_A)$

The Average Speed depends on the path.



Positive velocity
Zero velocity
Negative velocity
2
+1200
Δt = 400 s
Δx = 0 m
Δt = 400 s
+800
1
Δx = +400 m
Δx =
–400 m
3
Position x (m)
+400
Δt = 200 s
0
0
200
400
600
800
1000
1200
1400
1600
1800
Time t (s)

The spontaneous velocity



(b)

$V_{AF} = (X_F – X_A) / (t_F – t_A) < 0$

$V_{AE} = (X_E – X_A) / (t_E – t_A) < 0$

$V_{AD} = (X_D – X_A) / (t_D – t_A) < 0$

$V_{AC} = (X_C – X_A) / (t_C – t_A) = 0$

$V_{AB} = (X_B – X_A) / (t_B – t_A) > 0$

$V_A = \lim_{\Delta t \to 0} (X_{t+\Delta t} – X_t)/\Delta t$

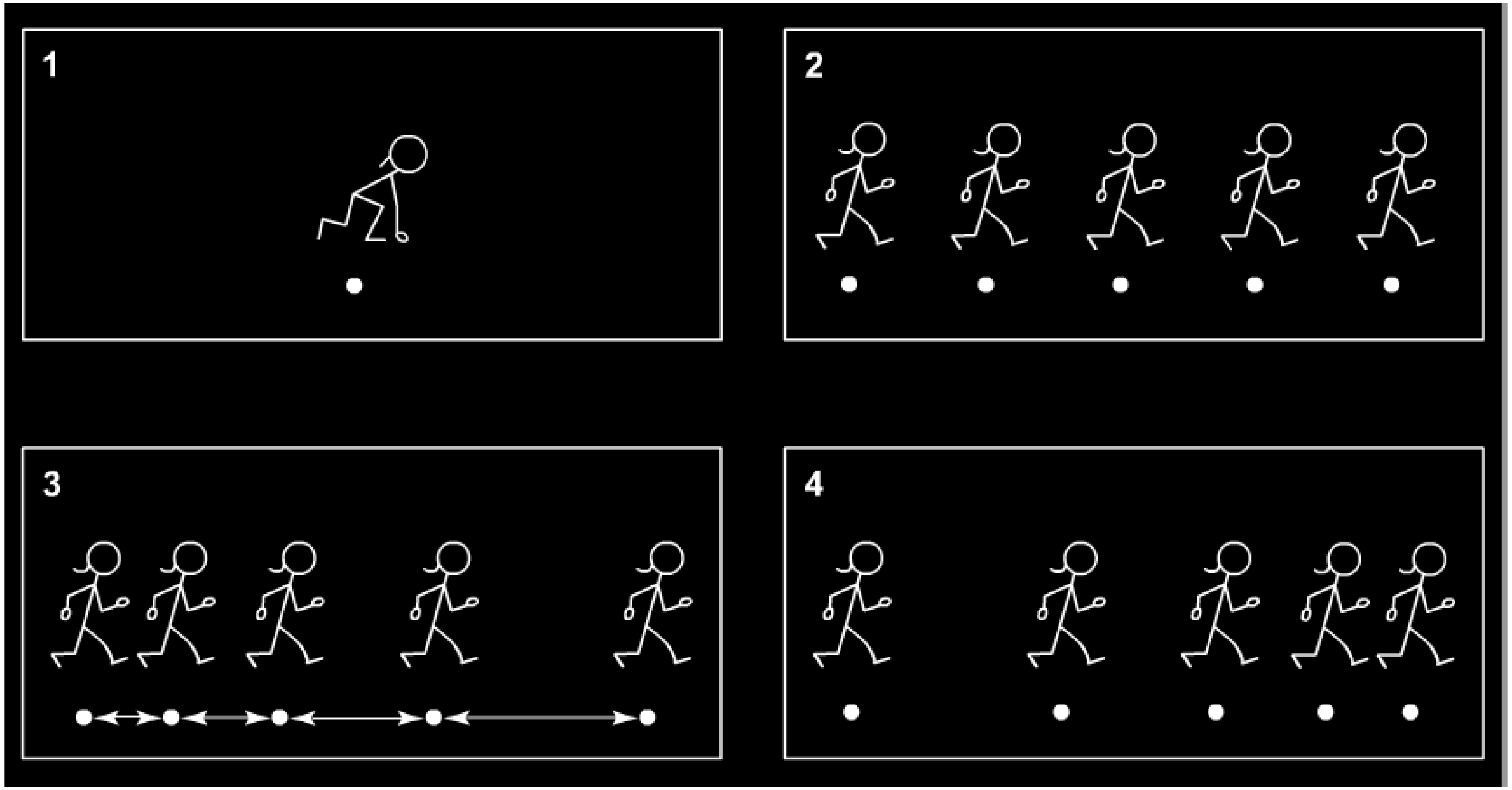
$= dX(t)/dt$

The spontaneous speed

$Speed_A = |V_A|$ → The Speed of a particle is ALWAYS positive, while its velocity can be both Positive or negative.

Ex. $V_A = -80$ m/s $V_B = 20$ m/s

→ $V_A < V_B$ $Speed_A > Speed_B$


1
2
3
4

Average acceleration

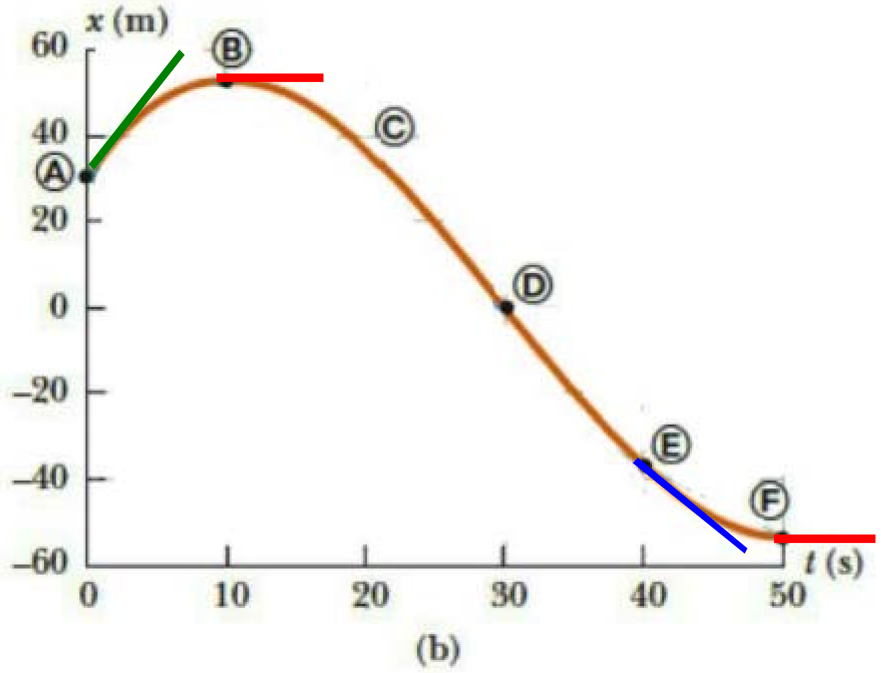
$\bar{a}_{t2,t1} = (V_2 - V_1)/(t_2 - t_1)$

$V_A > 0 \quad V_B = 0$

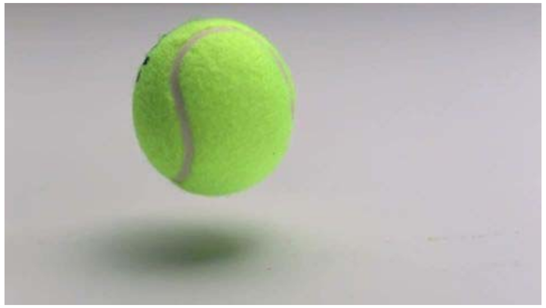
$\bar{a}_{AB} = (V_B - V_A)/(t_B - t_A) < 0$

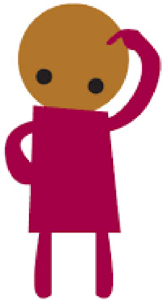
$V_E < 0 \quad V_F = 0$

$\bar{a}_{FE} = (V_F - V_E)/(t_F - t_E) > 0$



$\bar{a}_{BF} = (V_F - V_B)/(t_F - t_E) = 0$





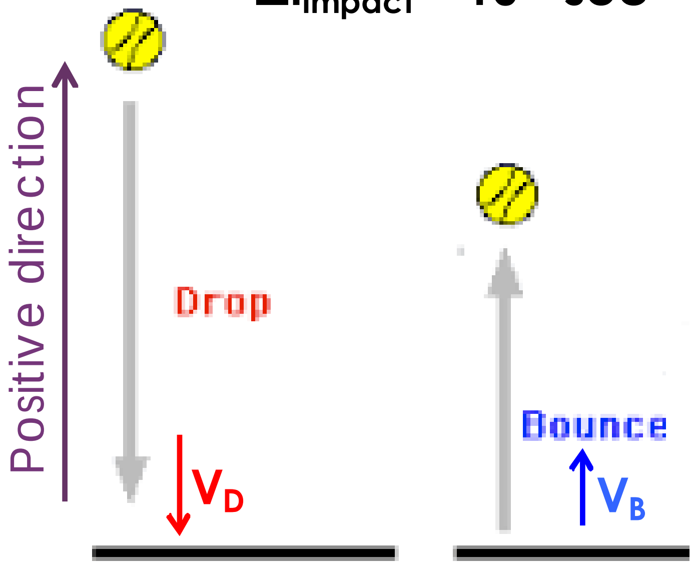
A tennis ball drops …

What's the impact acceleration?

1. Make a reasonable guess…

$|V_D| \approx |V_B| = 5$ m/s

$\Delta t_{impact} = 10^{-2}$ sec



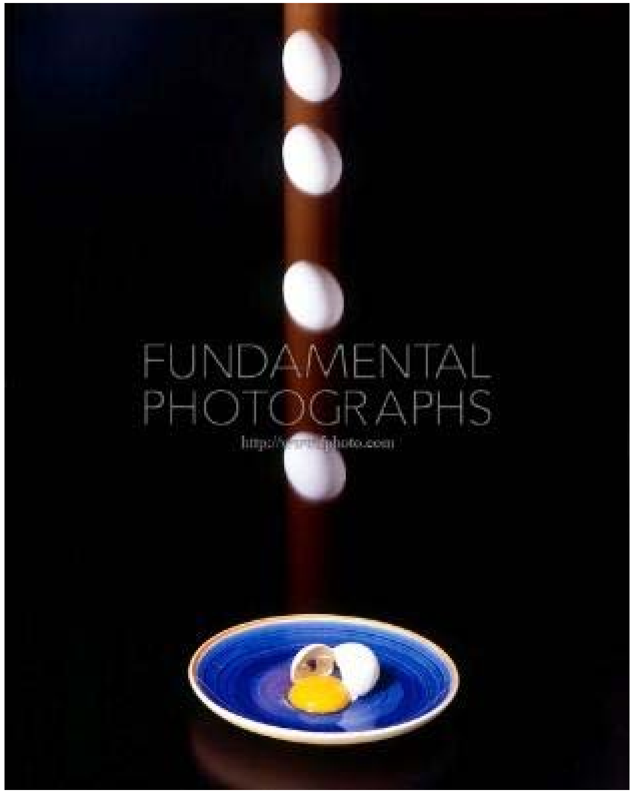
2. Choose a positive direction

3. $\bar{a}_{impact} = [V_B - V_D] / \Delta t$

$= [5-(-5)] / 10^{-2} = 10^3\ m/s^2$

What if we change the positive direction?

$\bar{a}_{impact} = [5-5] / 10^{-2} = -10^3$ m/s



Drop an egg and a tomato!

They will not bounce back…

What's the impact acceleration?

$|V_{drop}| \approx 5$ m/s , $V_{final} = 0$

$\Delta t_{impact} \approx 0.25$ s

$\bar{a}_{impact} = [V_{final} - V_{Drop}] / \Delta t = \pm 20$ m/s^2

The sign depends on the choose of positive direction

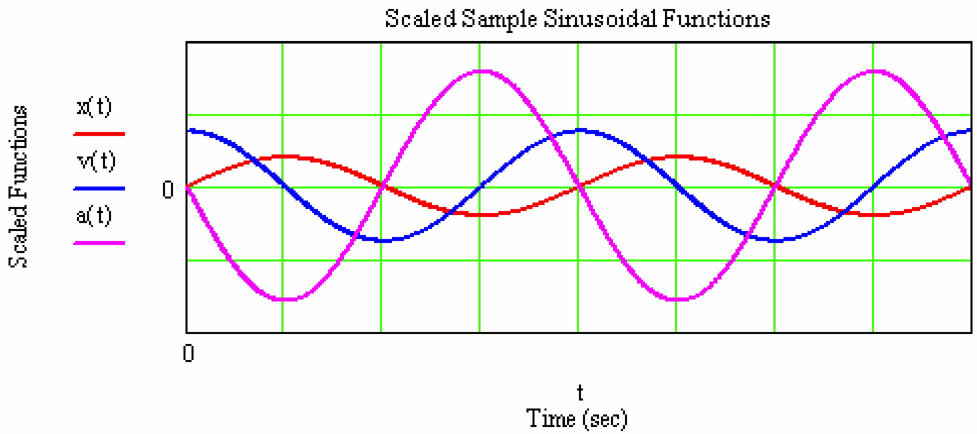
No matter what the sign would be, an egg can not tolerate such an acceleration and it would be smashed.

Spontaneous acceleration

$a_t = \lim_{\Delta t \to 0} (V_{t+\Delta t} - V_t)/\Delta t$

$= dV(t)/dt$

Figure 1 : Displacement, Velocity, and Acceleration Curves
Red curve = displacement = x(t)
Blue curve = velocity = u(t)
Magenta curve = acceleration = a(t)



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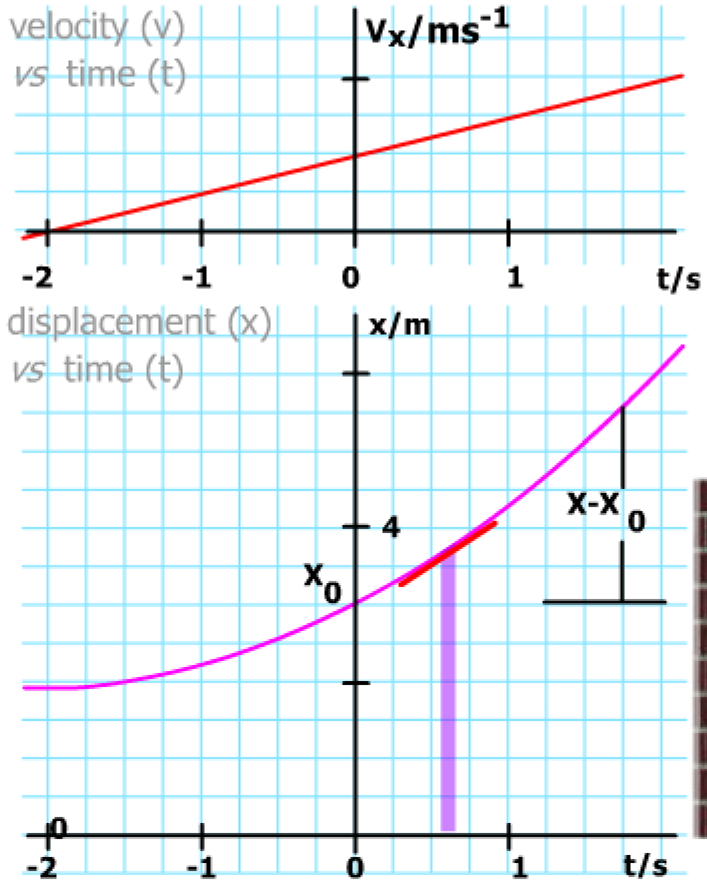
Ex.

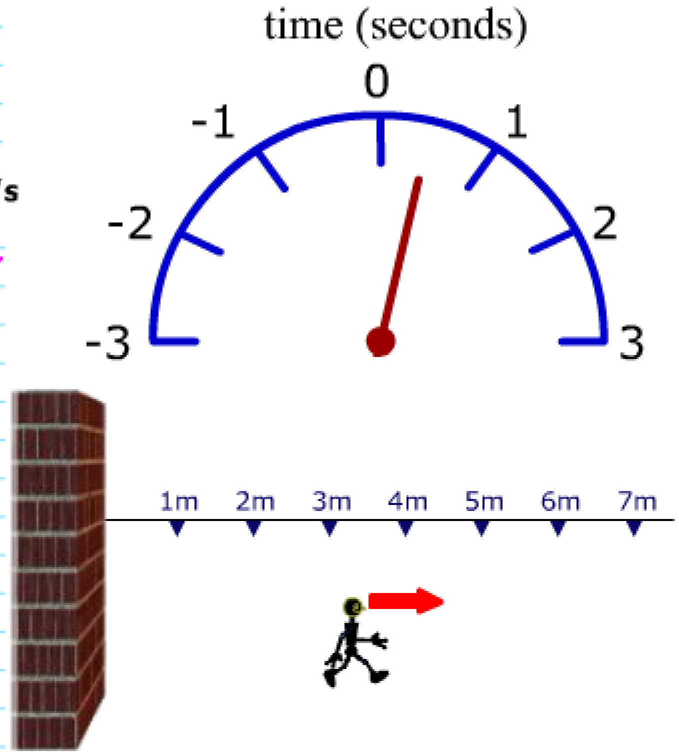
$X(t)=a\sin(\beta t)$

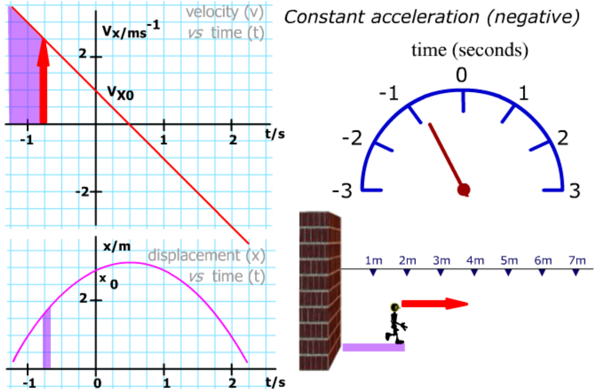
$V(t)=dX(t)/d(t)=a\beta\cos(\beta t)$

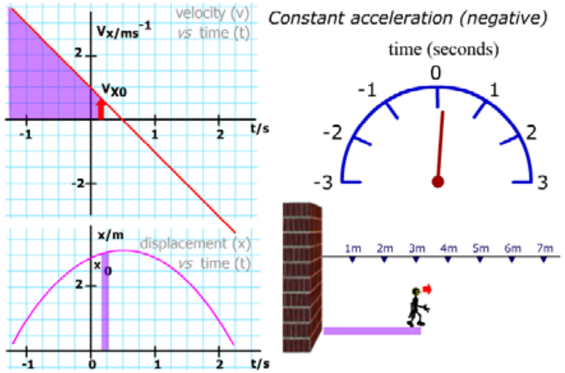
$a(t)=dV(t)/dt=-a\beta^2\sin(\beta t)$



Constant acceleration




Constant acceleration (negative)
velocity (v) vs time (t)
displacement (x) vs time (t)
time (seconds)
1m 2m 3m 4m 5m 6m 7m


Constant acceleration (negative)
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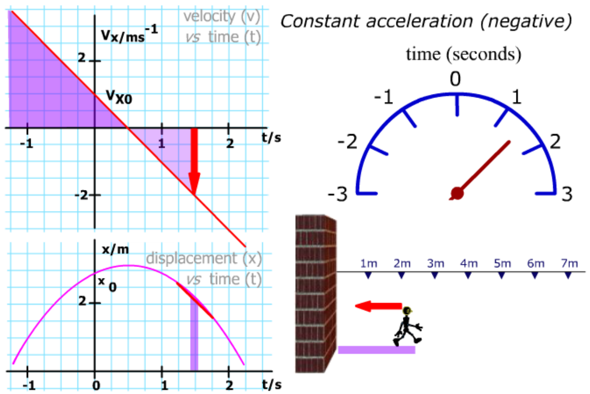
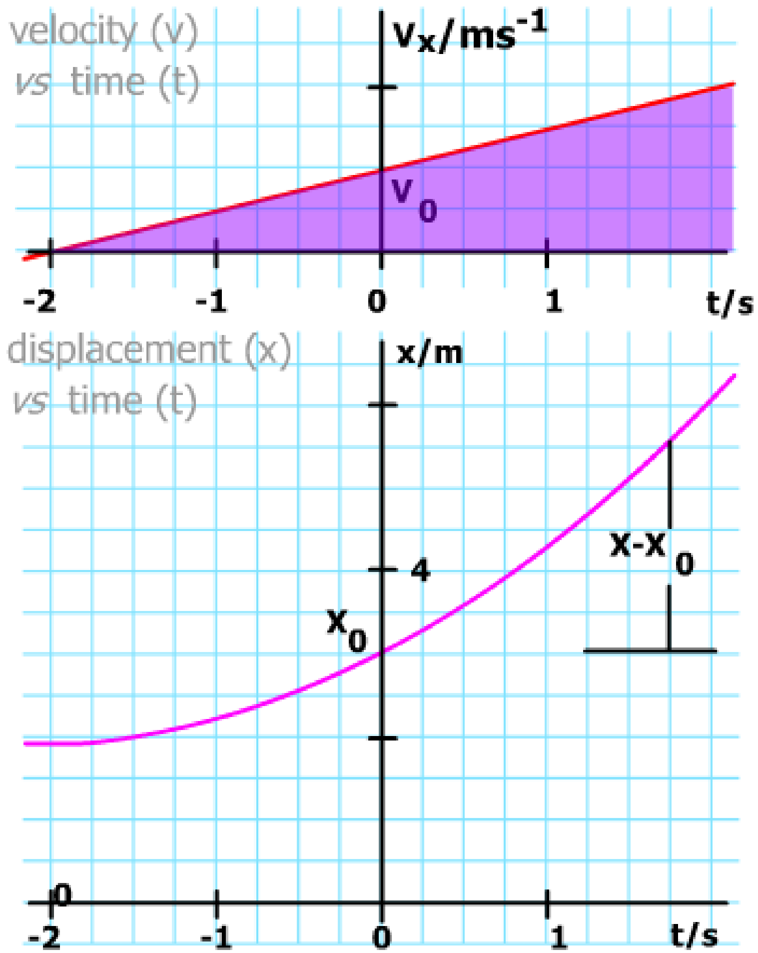

Constant acceleration (negative)
velocity (v) vs time (t)
displacement (x) vs time (t)
time (seconds)
1m 2m 3m 4m 5m 6m 7m

The equations of constant acceleration.



Constant acceleration

$$v = \int a\,dt = at + \text{constant}$$

$t = 0 \rightarrow v = v_0$ so:

$$v = v_0 + at \qquad \text{(i)}$$

$$x = \int v\,dt$$

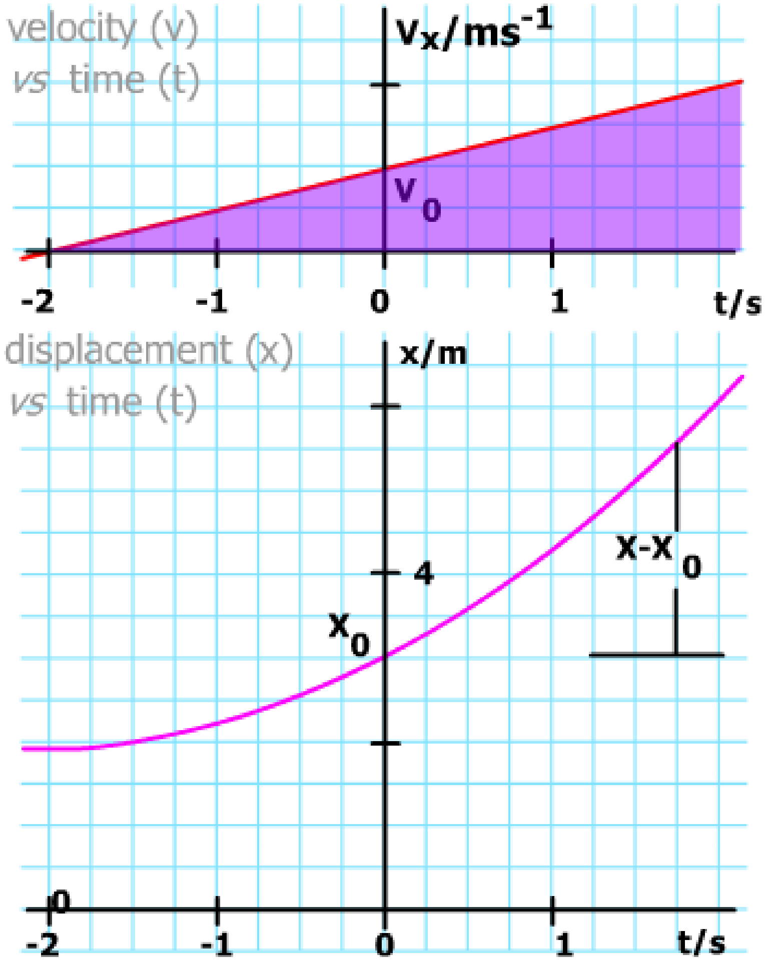
$$= \int v_0 + at\ dt$$

$$= v_0 t + \frac{1}{2}at^2 + \text{constant}$$

at $t = 0,\ x = x_0$ so:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad \text{(ii)}$$

The equations of constant acceleration.



Constant acceleration

$$v = \int a\,dt = at + \text{constant}$$

$$t = 0 \rightarrow v = v_0 \quad \text{so:}$$

$$v = v_0 + at \qquad \text{(i)}$$

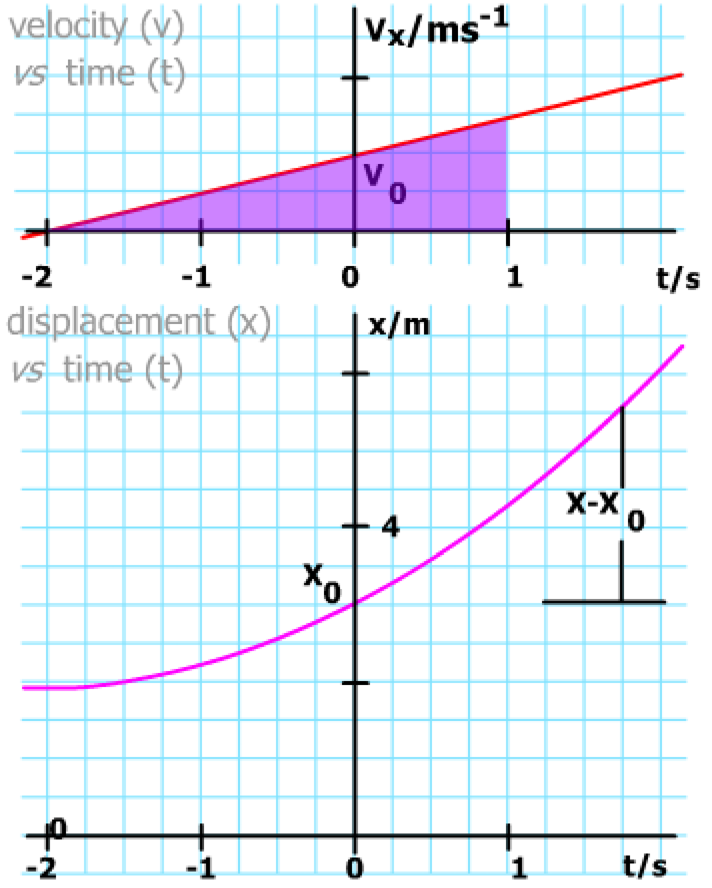
$$x = \int v\,dt$$

$$= \int v_0 + at\ dt$$

$$= v_0 t + \frac{1}{2}at^2 + \text{constant}$$

$$\text{at } t = 0,\ x = x_0 \quad \text{so:}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad \text{(ii)}$$



Constant acceleration

Definition of constant a →

$$v = v_0 + at \qquad \text{(i)}$$

Integration of (i) →

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad \text{(ii)}$$

Eliminate t →

$$2a(x - x_0) = v^2 - v_0^2 \qquad \text{(iii)}$$

$V(t) = at + V_0$

$V(t_i) = V_i = at_i + V_0$

$V(t_f) = V_f = at_f + V_0$

$V_f - V_i = a(t_f - t_i)$

$V_f = a\Delta t + V_i$

$V(t) = at + V_0 \quad t = (V - V_0)/a$

$X(t) = 1/2at^2 + V_0t + X_0$

$\rightarrow X(t) = 1/2a[(V - V_0)/a]^2 + V_0(V - V_0)/a + X_0$

$= (V^2 - V_0^2)/2a + X_0$

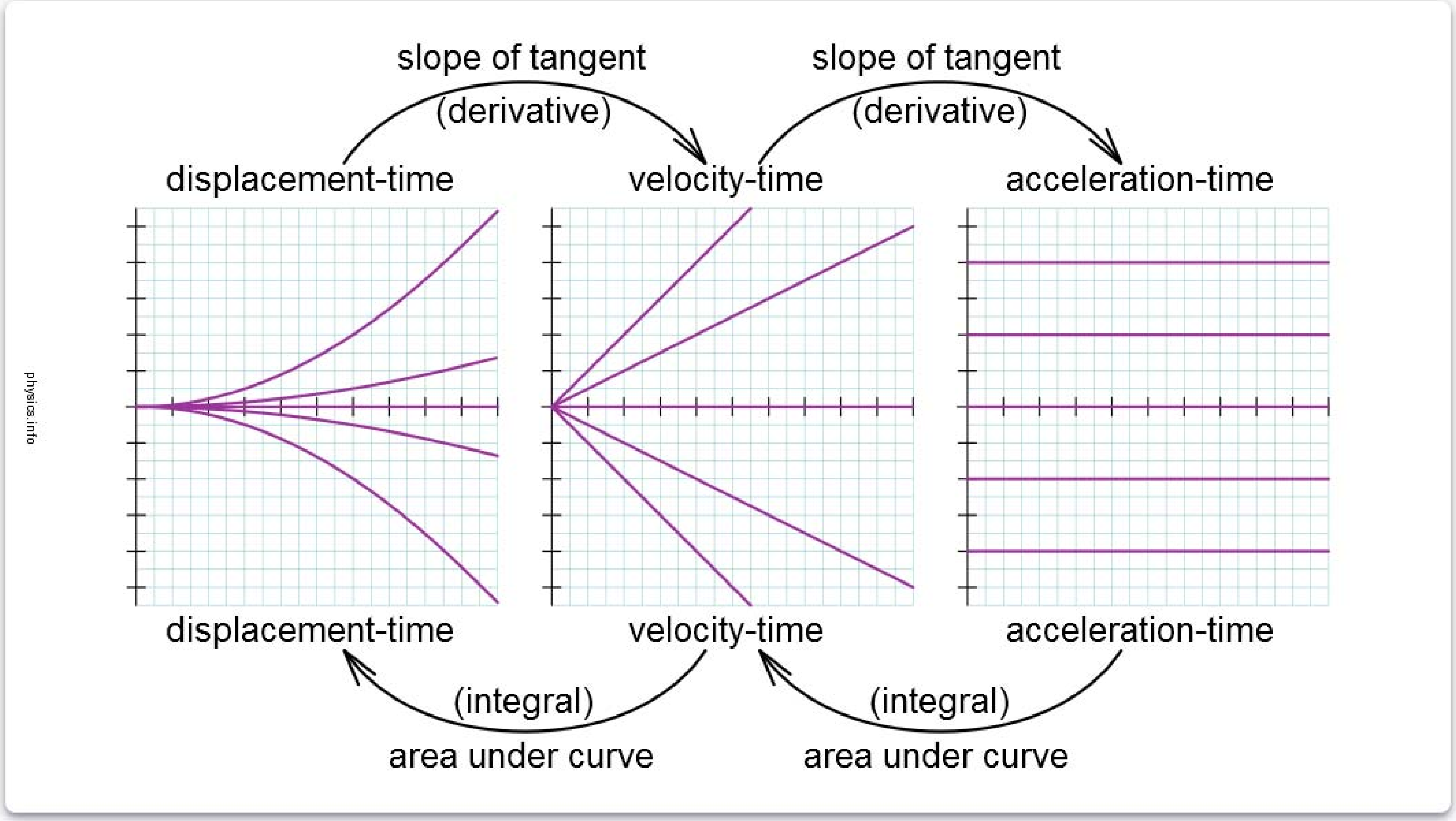
$V^2 = V_0^2 + 2a\Delta X$

$V(t) = at + V_0 \quad a = (V - V_0)/t$

$X(t) = 1/2at^2 + V_0t + X_0$

$\rightarrow X(t) = 1/2[(V - V_0)/t]t^2 + V_0t + X_0$

$X(t) = (V + V_0)t/2 + X_0$



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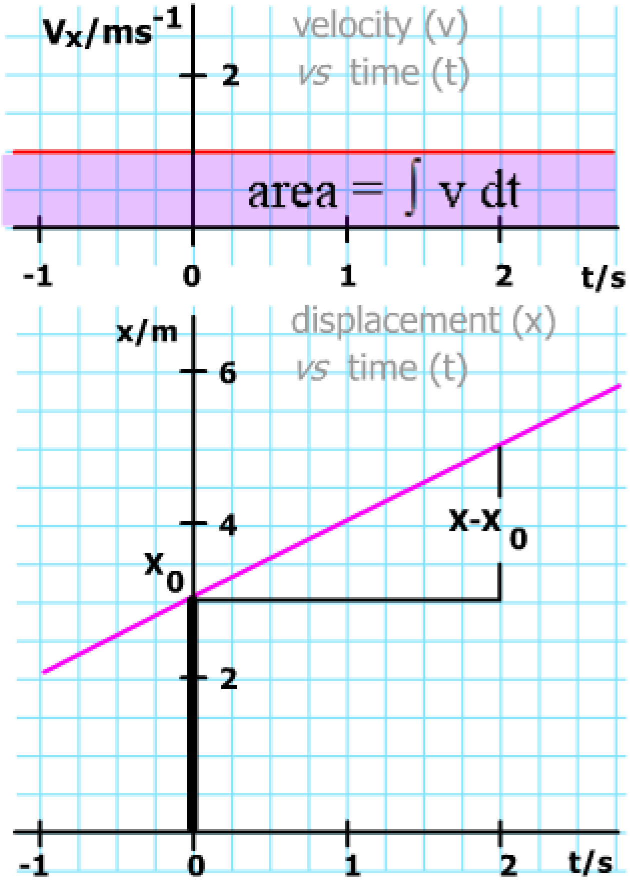
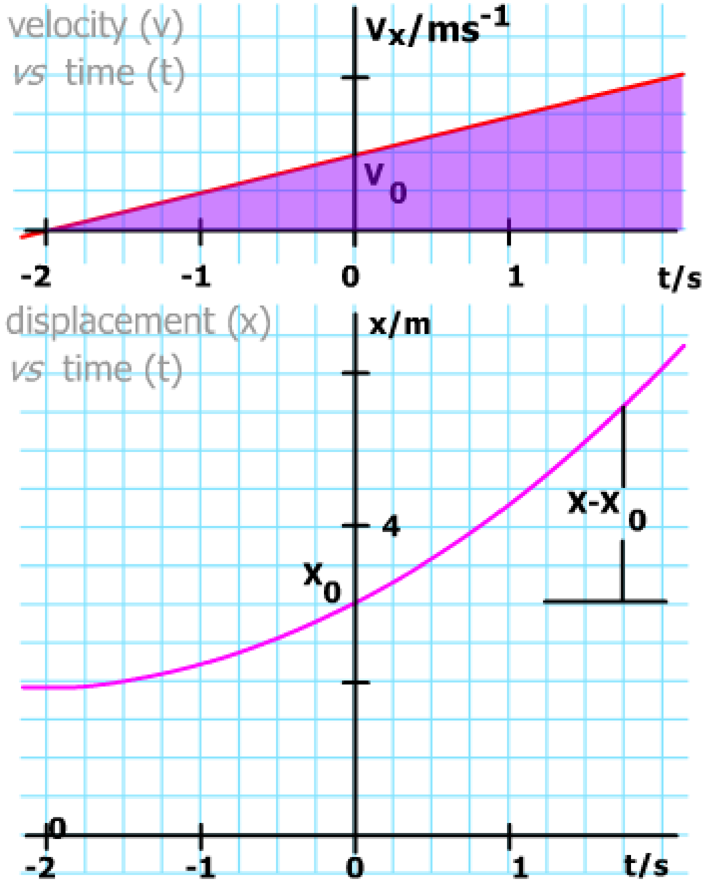


V_x/ms^{-1}
velocity (v)
vs time (t)
2
area = $\int$ v dt
-1
0
1
2
t/s
x/m
displacement (x)
vs time (t)
6
4
2
x_0
$x-x_0$
-1
0
1
2
t/s



velocity (v)
vs time (t)
V_x/ms^{-1}
v_0
-2
-1
0
1
t/s
displacement (x)
vs time (t)
x/m
4
x_0
$x-x_0$
0
-2
-1
0
1
t/s

So, Which equation should I use???

$X(t)=1/2at^2+V_0t+X_0$

$V(t)=at+V_0$

$V_f=a\Delta t+V_i$

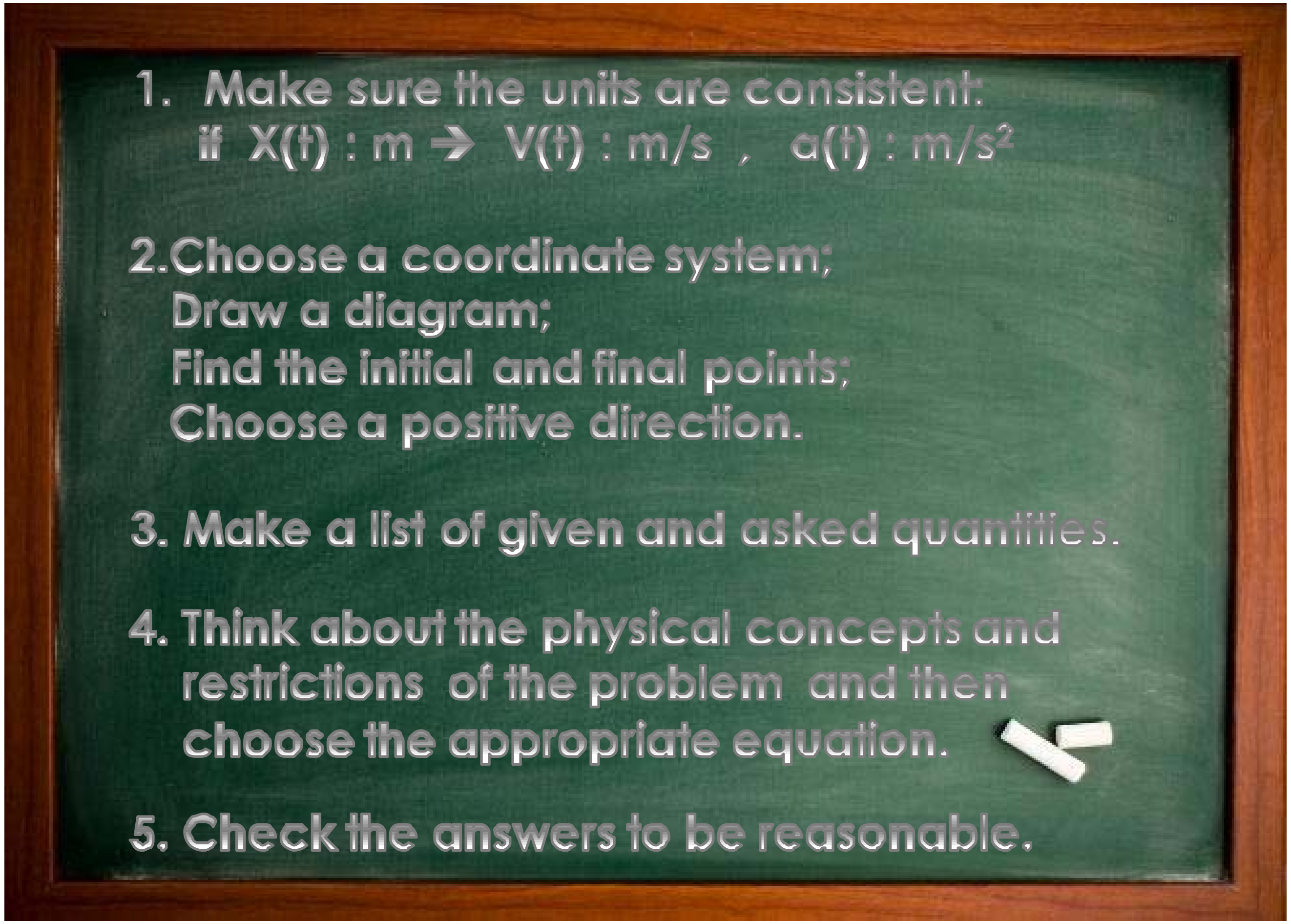
$V^2=V_0^2+2a\Delta X$

$X(t)=(V+V_0)t/2+X_0$

It depends on the problem.

Included quantities: (given/asked)	Not included quantities:	Appropriate equation:
$X(t)$, a , t , V_0 , X_0	$V(t)$	$X(t)=1/2at^2+V_0t+X_0$
$V(t)$, a , t , V_0	$X(t)$, X_0	$V(t)=at+V_0$
V_f , V_i , a , Δt	$X(t)$, X_0	$V_f=a\Delta t+V_i$
$\Delta X(t)$, a , V , V_0	t	$V^2=V_0^2 +2a\Delta X$
$X(t)$, $V(t)$, t , V_0 , X_0	a	$X(t)=(V+V_0)t/2+X_0$

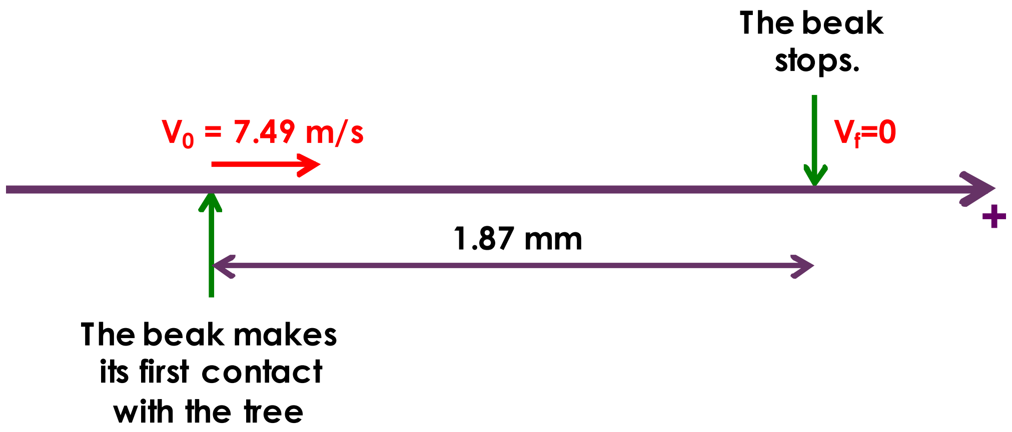
Before starting to solve make sure you've checked following points:



Find a woodpecker's hamming acceleration :

Contact speed : 7.49 m/s
Penetration length : 1.87 mm

$X = 1.87 \text{ mm} = 1.87 \times 10^{-3} \text{ m}$



Given quantities	Asked quantity	Appropriate equation
V_0 , V_f , ΔX	a	$V^2 = V_0^2 + 2a\Delta X$

$0 = 7.49^2 + 2\ a \times 1.87 \times 10^{-3}$ ➔ $a = -1.5 \times 10^4\ m/s^2$

Make a list of given and asked Quantities. Think about the physical

Choose a coordinate system:

Draw a diagram;

Find the initial and final points;

Choose a positive direction.

How can the woodpecker withstand such a huge acceleration?

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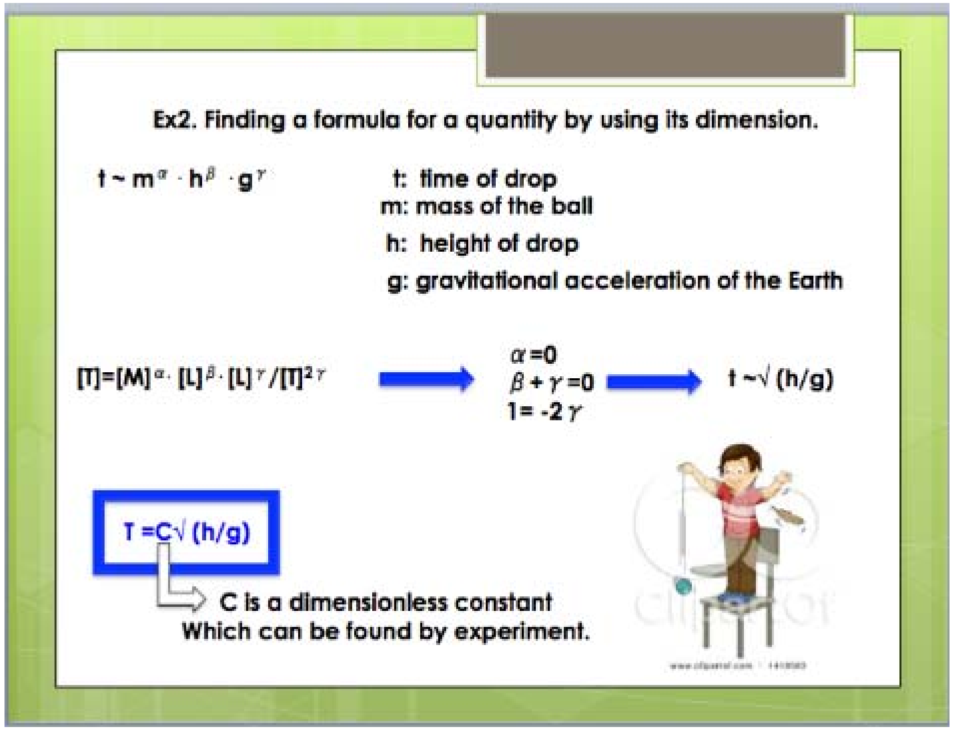
Free Fall : a constant acceleration motion.

$|a| = g = 9.8\ m/s^2$



By the dimensional analysis, we found out that the the time of drop for free fall is independent of the mass
We are going to reach the same result by <u>the equations of motion.</u>

Free Fall : a constant acceleration motion.

$|a| = g = 9.8\ m/s^2$

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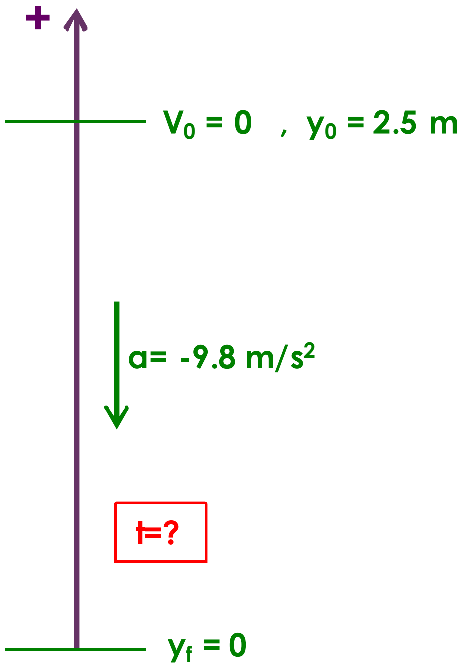
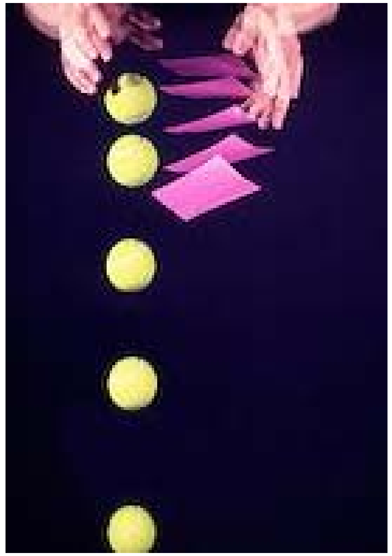


$$y(t) = 1/2at^2 + V_0t + y_0$$

$$0 = -9.8/2\ t^2 + 0 + 2.5$$

$$t = 0.714\ \text{s}$$





38 You are driving toward a traffic signal when it turns yellow. Your speed is the legal speed limit of u_0 : 55 km/h; your best ***deceleration* rate has the ***magnitude*** a = 5.18 m/s². Your best reaction time to begin braking is T = 0.75 s. To avoid having the front of your car enter the intersection after the light turns red, should you brake to a stop or continue to move at 55 km/h if the distance to the intersection and the duration of the yellow light are (a) 40 m and 2.8 s, and (b) 32 m and 1.8 s? Give an answer of brake, continue, either (if either strategy works), or neither (if neither strategy works and the yellow duration is inappropriate).

V_0 = 55km/h = 55×1000/3600 m/s ≅15.27 m/s

a_{break} = a_b = −5.18 m/s^2

$T_{reaction}$ = T_r = 0.75 s

$D_{intersection}$=D_i = (a) 40m , (b) 32 m

$T_{yellowlight}$ = T_y = (a)2.8s , (b)1.8 s

If you break:

Constant Speed — REACTION: V_0 → V_0, $D_r = ?$, T_r

Constant Acceleration — BREAK: V_0 → $V=0$, $D_b = ?$, a_b

Reaction:

$x = vt + x_0$ $\qquad$ $x - x_0 = D_r = V_0 T_r$

Breaking:

$V^2 - V_0^2 = 2\,a\,\Delta X$ $\qquad$ $0 - v_0^2 = 2a_b D_b$ $\qquad$ $D_b = -v_0^2/2a_b$

$D_{total} = D_r + D_b$

a) $D_t \cong 34$ m $< D_{intersection} = 40$ m it can break.

b) $D_t \cong 34$ m $> D_{intersection} = 32$ m it can Not break.

If you continue to drive:

Constant Speed

CONTINUE

V_0 $\longleftrightarrow$ V_0

$D_{intersection} = D_i$

$T_{intersection} = T_i = ?$

Intersection:

$x = vt + x_0$ $\quad$ $x - x_0 = D_i = v_0 T_i$ $\quad$ $T_i = D_i / v_0$

a) $T_i \cong 2.61$ s $< T_{yellow} = 2.8$ s it can pass.

b) $T_i \cong 2.09$ s $> T_{yellow} = 1.8$ s it can Not pas.

Chapter 1:
problems
#11 #17

Chapter 2 :
problems
#12 #30 #40