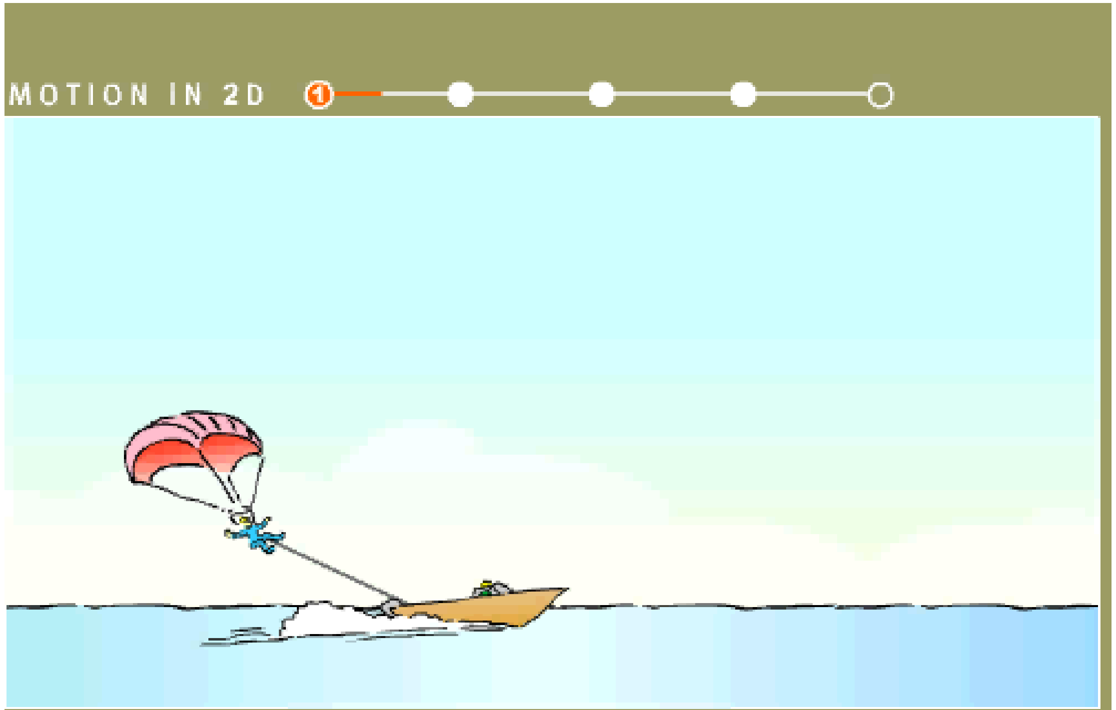
Motion in two and three dimensions. (Part I)

Meghdadi Fall 2016



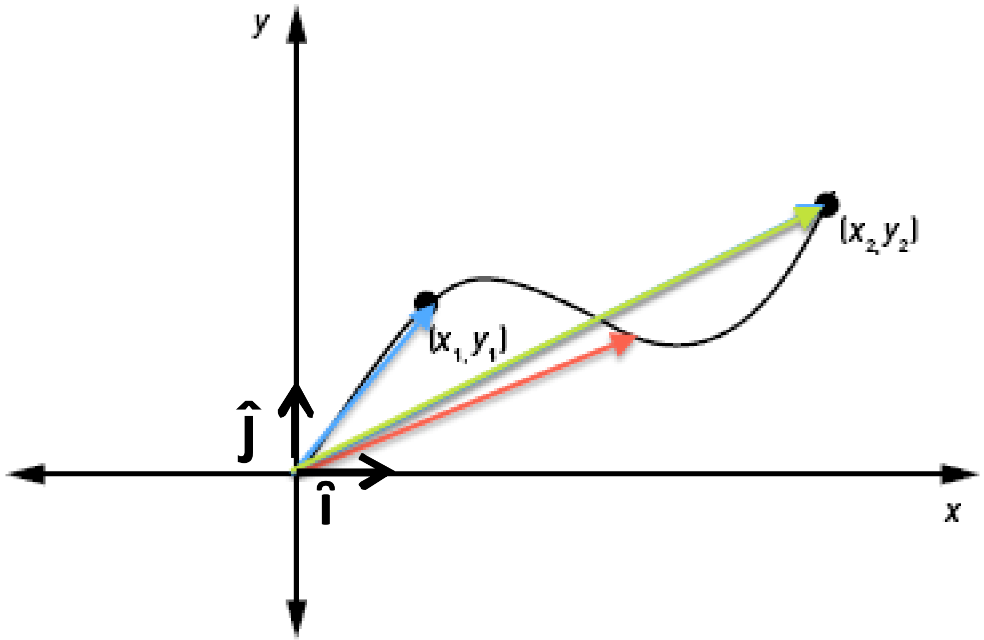
To describe motion in two dimensions, we use vectors.

$r_1(t) = (x_1(t)\ ,\ y_1\ (t)\ ) = x_1\ \hat{i} + y_1\ \hat{j}$

$r(t)\ = (\ x(t)\ ,\ y\ (t)\ ) = x\ \hat{i} + y\ \hat{j}$

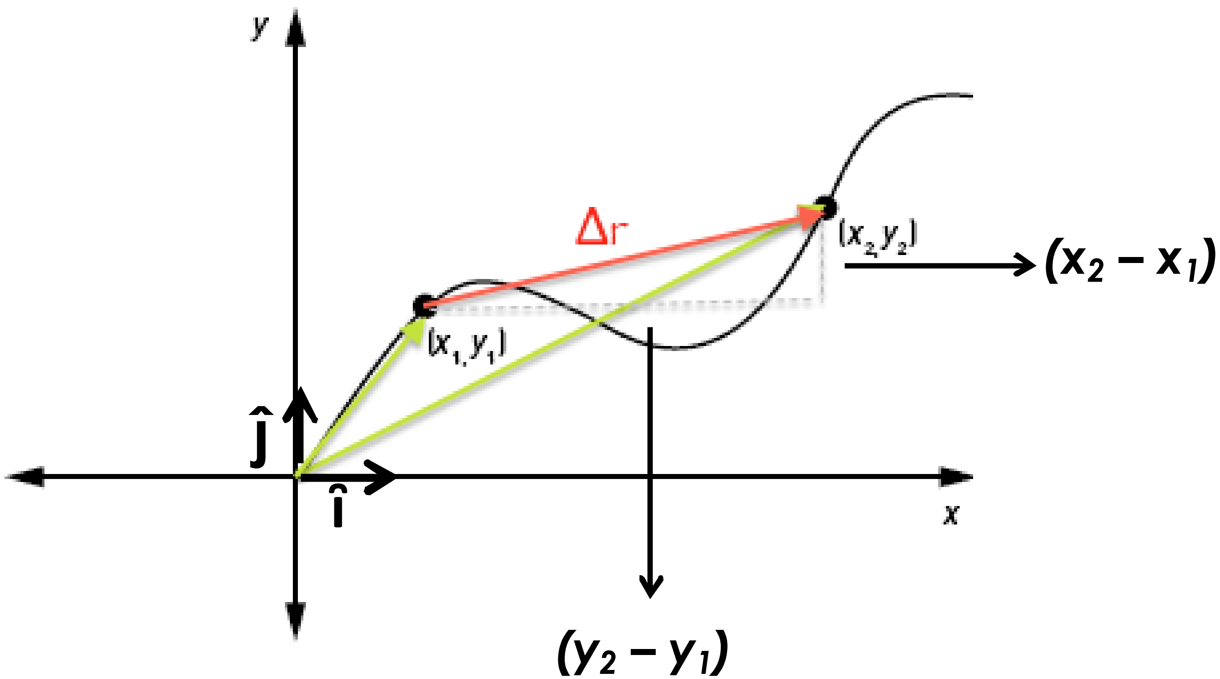
$r_2(t) = (x_2(t)\ ,\ y_2\ (t)\ ) = x_2\ \hat{i} + y_2\ \hat{j}$

- Position Vector
- Displacement Vector

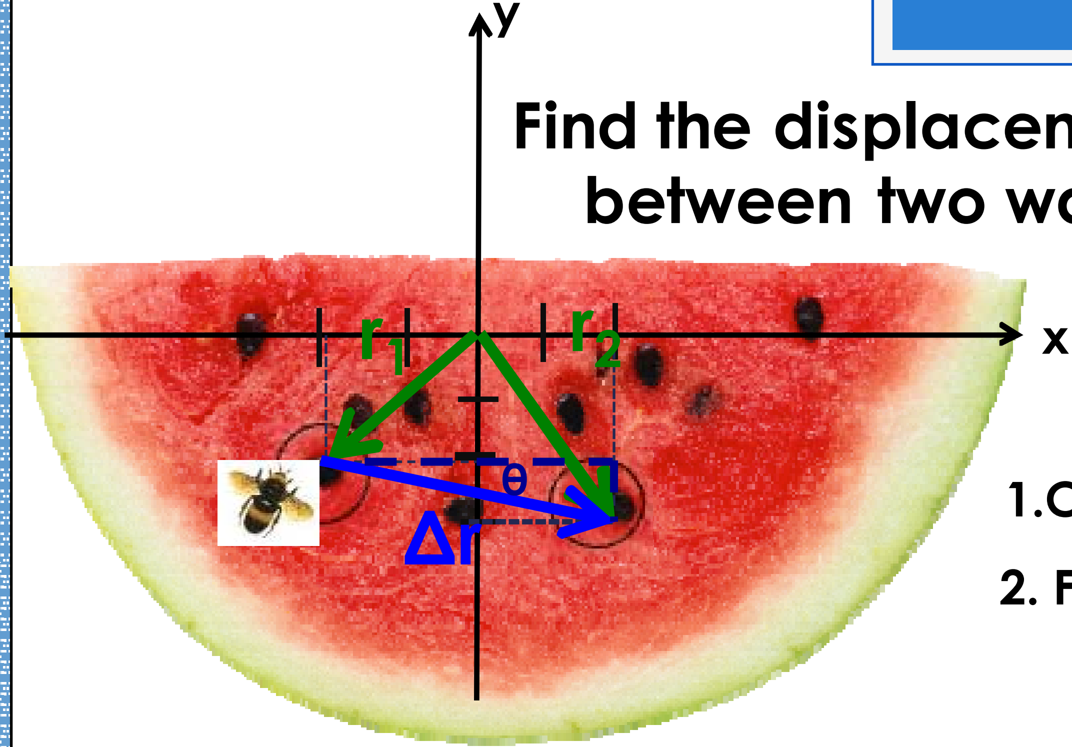


$\Delta r = r_2 - r_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$= \Delta x \quad \hat{i} + \Delta y \quad \hat{j}$



Find the displacement vector of a bee between two watermelon's seeds.



1. Choose a coordinate system.
2. Find the position vectors.

$r_1 = -2\,\hat{\imath} - 2\,\hat{\jmath}$

$r_2 = 2\,\hat{\imath} - 3\,\hat{\jmath}$

→ $r_2 - r_1 = (2-(-2))\,\hat{\imath} + (-3-(-2))\,\hat{\jmath}$ → $\Delta r = 4\,\hat{\imath} - \hat{\jmath}$

$|\Delta r| = \sqrt{(4^2 + 1)}$ → $|\Delta r| = \sqrt{17}$

$\tan\theta = 1/4$ → $\theta = \tan^{-1} 1/4 \cong 14^o$

Velocity Vector

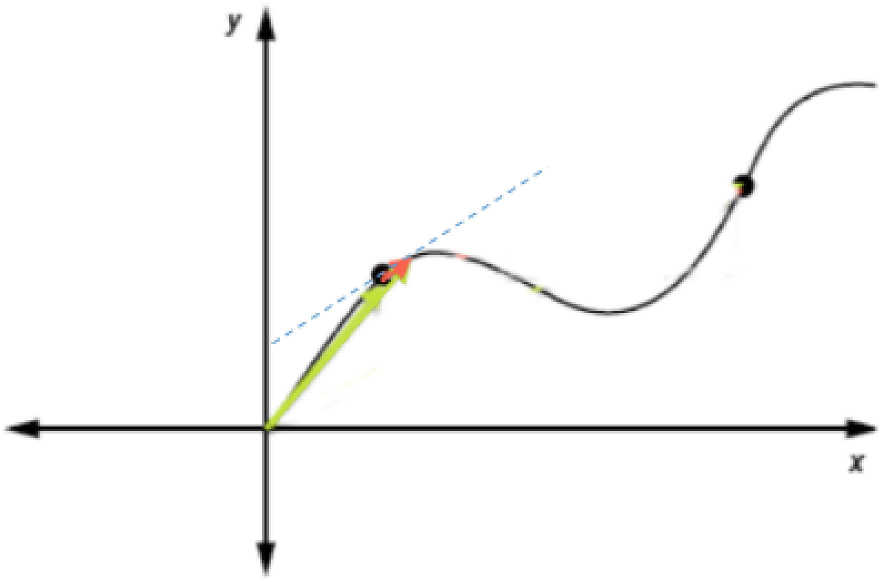
- Average velocity
- Instantaneous velocity

$V_{av} = \Delta r / \Delta t$

$V_{in} = \lim_{\Delta t \to 0} \Delta r / \Delta t = dr / dt$

$= d(x\,\hat{i} + y\,\hat{j} + z\,\hat{k})/dt$

$= V_x\,\hat{i} + V_y\,\hat{j} + V_z\,\hat{k}$



The direction of V_{in} is always tangent to the particle's path.

Problem10:

The position vector $\underline{r=5.00\,t\,\hat{i} + (e\,t + f\,t^2)\,\hat{j}}$ locates a particle as a function of time. Vector r is in meters, t is in seconds, and factors e and f are constants. Figure 4-34 gives ***the angle of the particle's direction of travel as a function of t*** (measured from the positive x direction). What are (a) e , and (b) f, including units?

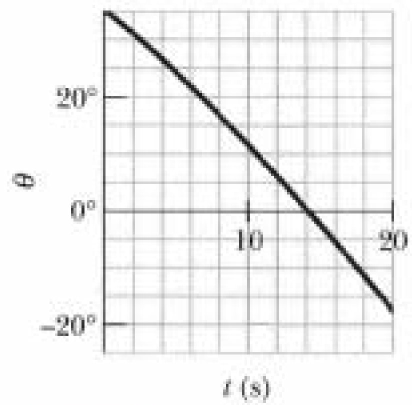


FIG. 4-34 Problem 10.

$[y(t)] = [e\,t + f\,t^2]$
$= [et] + [f\,t^2]$
$= L$

$\Rightarrow$

$[et] = L$

$[e]\,[t] = [e]\,T = L$

$\boxed{[e] = L/T}$

$[ft^2] = L$

$[f]\,[t]^2 = [f]\,T^2 = L$

$\boxed{[f] = L/T^2}$

Problem10:

The direction of V_{in} is always tangent to the particle's path.

The position vector *r=5.00 t î + (e t + f t²) ĵ* locates a particle as a function of time. Vector r is in meters, t is in seconds,and factors e and f are constants. Figure 4-34 gives *the angle of the particle's direction of travel as a function of t* (measured from the positive x dtection). What are (a) e , and (b) f, including units?

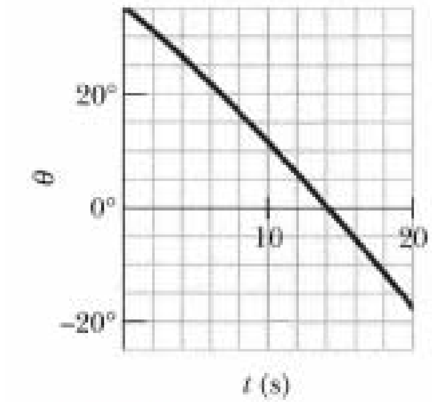


FIG. 4-34 Problem 10.

[e] = m/s

[f] = m/s^2

V_{in} = dr /dt
=d[5.00 t î + (e t + f t²) ĵ]/ dt
= 5.00 î + (e + 2 f t) ĵ

tan θ(t)= $V_y(t)$ / $V_x(t)$
= (e + 2 f t)/5.00

t= 0
tanθ(0) = tan 35^0 =[e +2f (0)]/5

e= 3.5 m/s

t= 14 s
tanθ(14) = tan 0^0 =e +2f (14)

f= - 0.125 m/s^2

Acceleration Vector

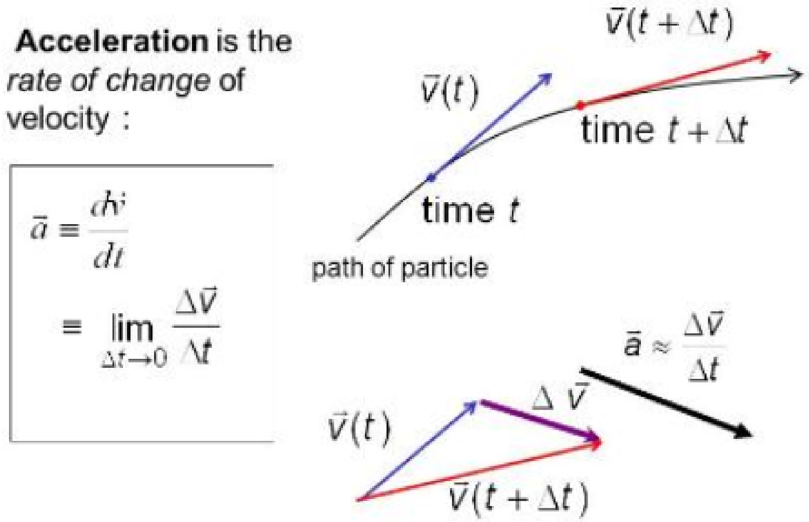
- Average acceleration
- Instantaneous acceleration

$a_{av}=\Delta V/\Delta t$

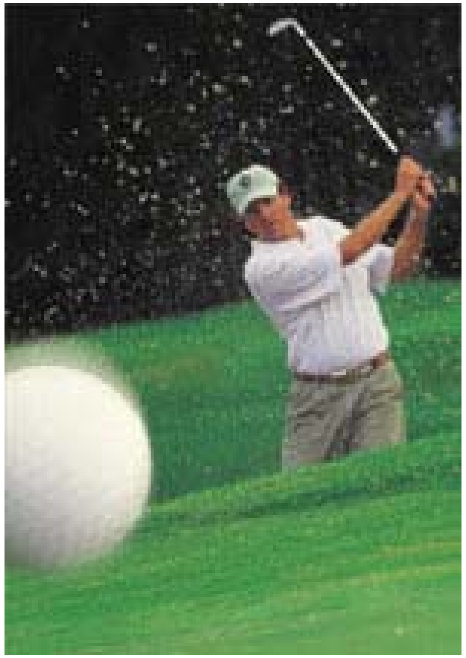
$a_{in}=\lim_{\Delta t\to 0}\Delta V/\Delta t = dV/dt$

$=d(V_x\hat{i} + V_y\hat{j} + V_z\hat{k})/dt$

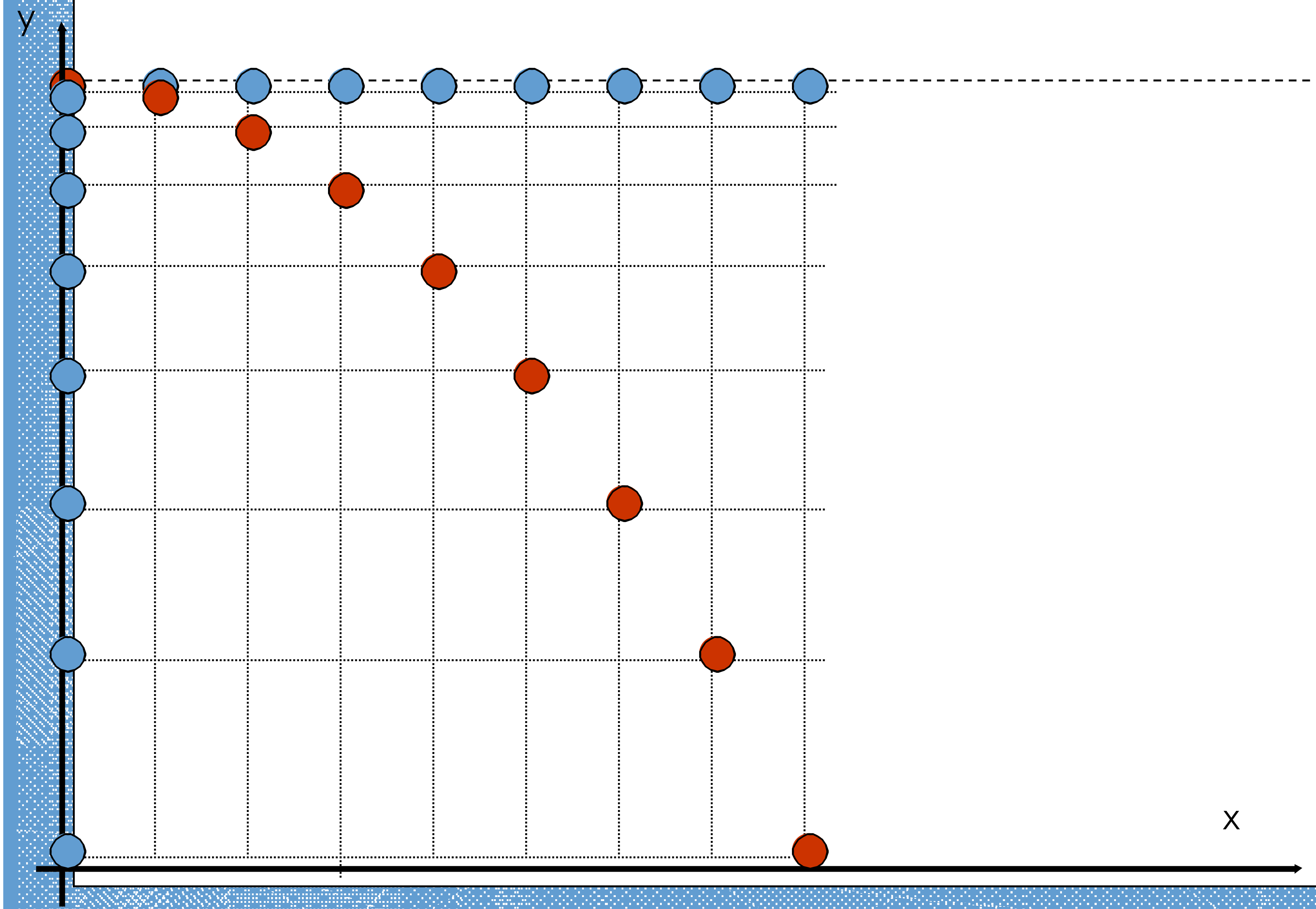
$=a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$



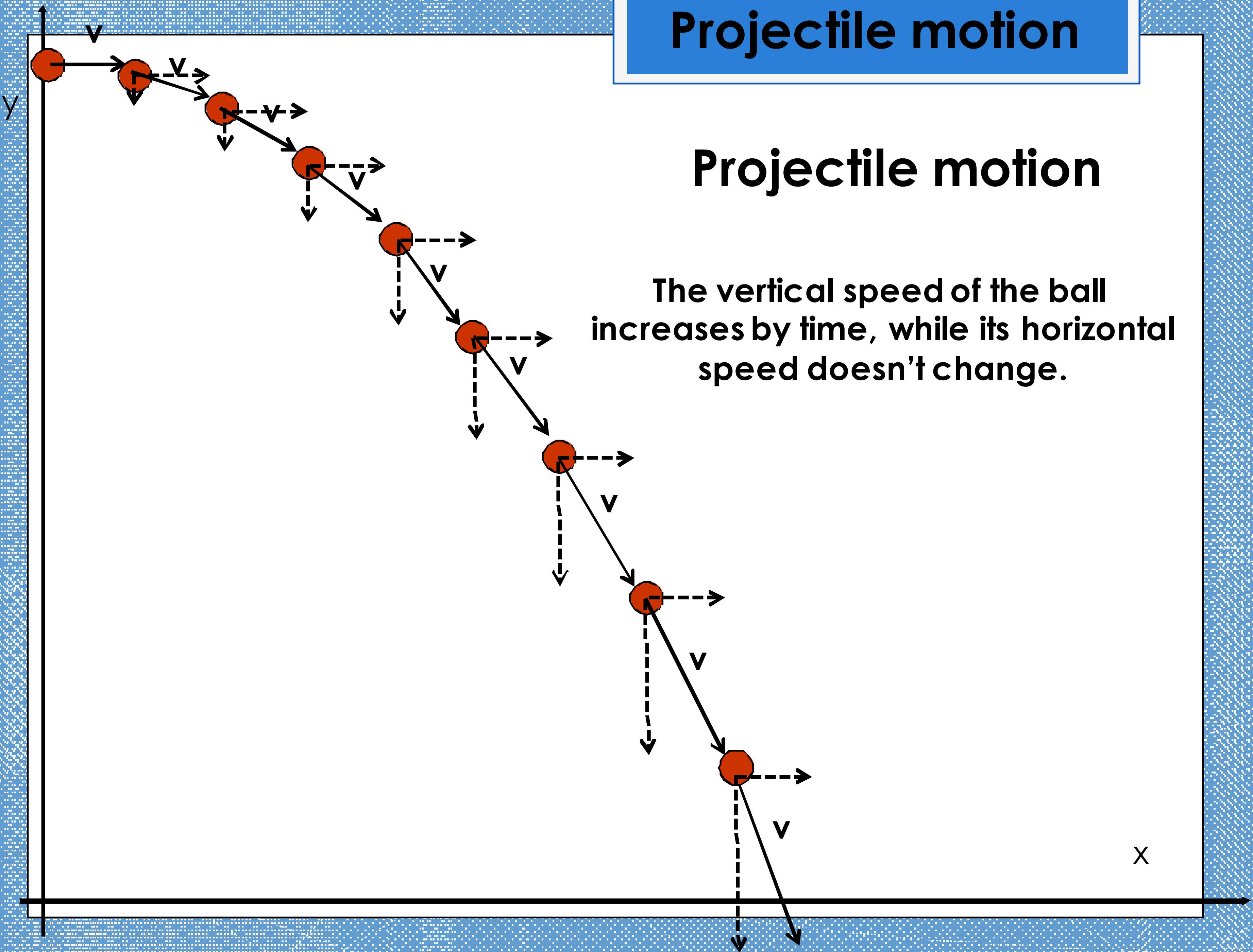
Projectile motion



Projectile motion



Projectile motion

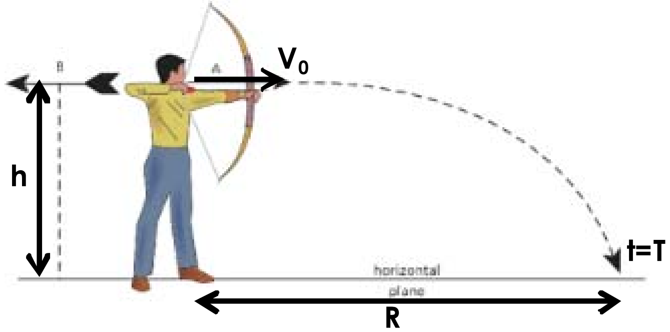


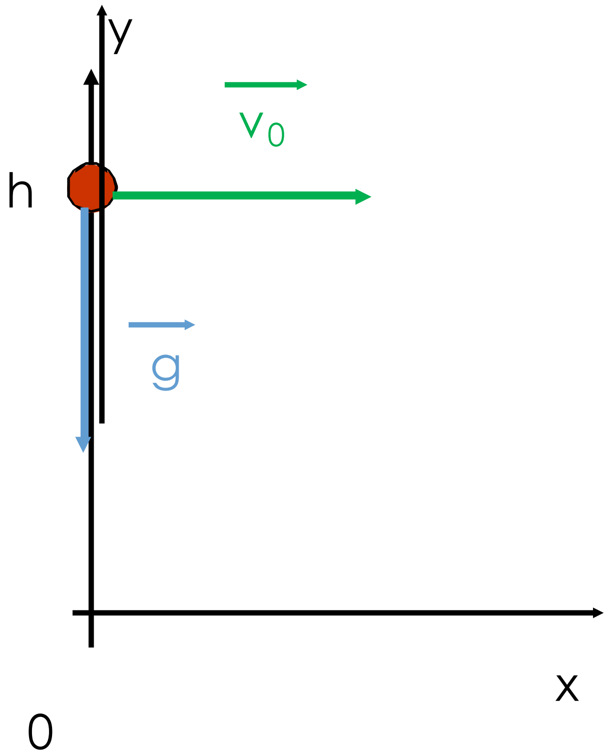
Projectile motion

The vertical speed of the ball increases by time, while its horizontal speed doesn't change.

Projectile motion

- Horizontal projectile ($\theta = 0$)

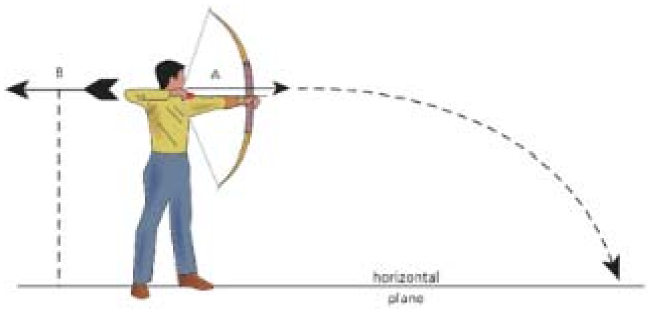




	X Uniform m.	Y Accel. m.
ACCL.	$a_x = 0$	$a_y = g = -9.81\ m/s^2$
VELC.	$v_x = v_0$	$v_y = g\ t$
DSPL.	$x = v_0 t$	$y = h + ½\ g\ t^2$

$$y = h + \frac{1}{2} g t^2$$

$$0 = h + \frac{1}{2} g (\Delta t)^2$$

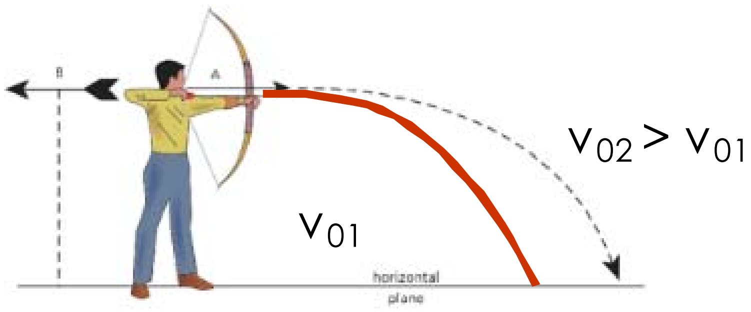


$$\Delta t = \sqrt{2h/(-g)}$$

$$\Delta t = \sqrt{2h/(9.81 ms^{-2})}$$

$$x = v_0\, t$$

$$y = h + ½\, g\, t^2$$



$t = x/v_0$

$y = h + ½\, g\, (x/v_0)^2$

$$y = h + ½\, (g/v_0^2)\, x^2$$

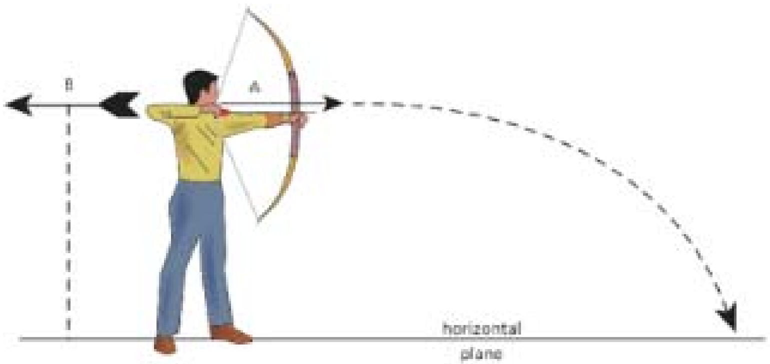
$$y = ½\, (g/v_0^2)\, x^2 + h$$

$$x = v_0 t$$

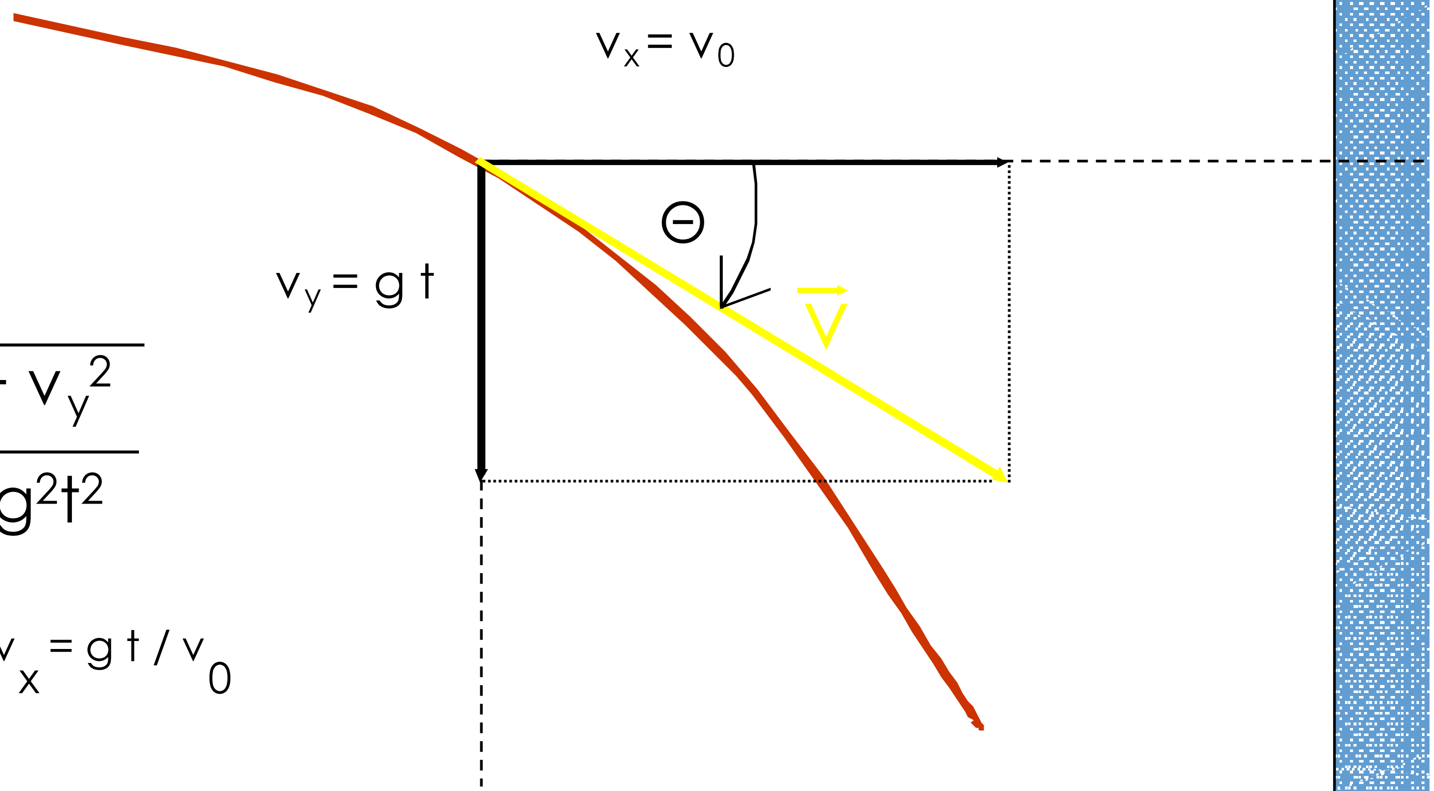
$$\Delta x = v_0 \Delta t$$

$$\Delta t = \sqrt{2h/(-g)}$$

$$\Delta x = v_0 \sqrt{2h/(-g)}$$



Δx



$v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{v_0^2 + g^2 t^2}$

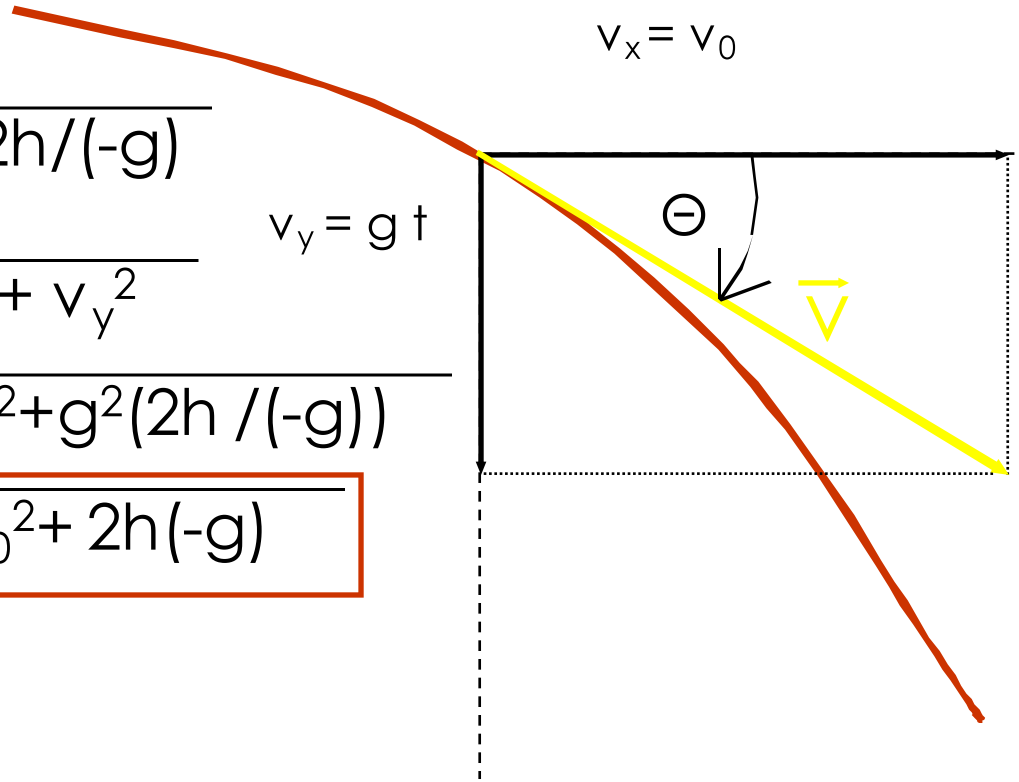
$tg\,\Theta = v_y / v_x = g\,t / v_0$

$$\Delta t = \sqrt{2h/(-g)}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v = \sqrt{v_0^2 + g^2(2h/(-g))}$$

$$\boxed{v = \sqrt{v_0^2 + 2h(-g)}}$$



$$\operatorname{tg}\Theta = g\,\Delta t / v_0$$

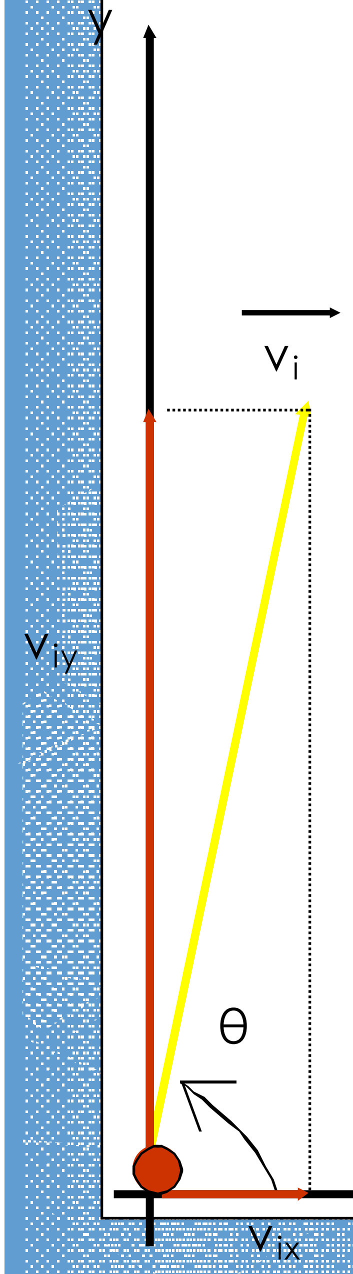
$$= -(-g)\sqrt{2h/(-g)} / v_0$$

$$= -\sqrt{2h(-g)} / v_0$$

$$\Theta < 0$$

$\Delta t = \sqrt{2h/(-g)}$
$\Delta x = v_0 \sqrt{2h/(-g)}$
$v = \sqrt{v_0^2 + 2h(-g)}$ $\mathrm{tg}\ \Theta = -\sqrt{2h(-g)}\ / \ v_0$

Lec3: 46

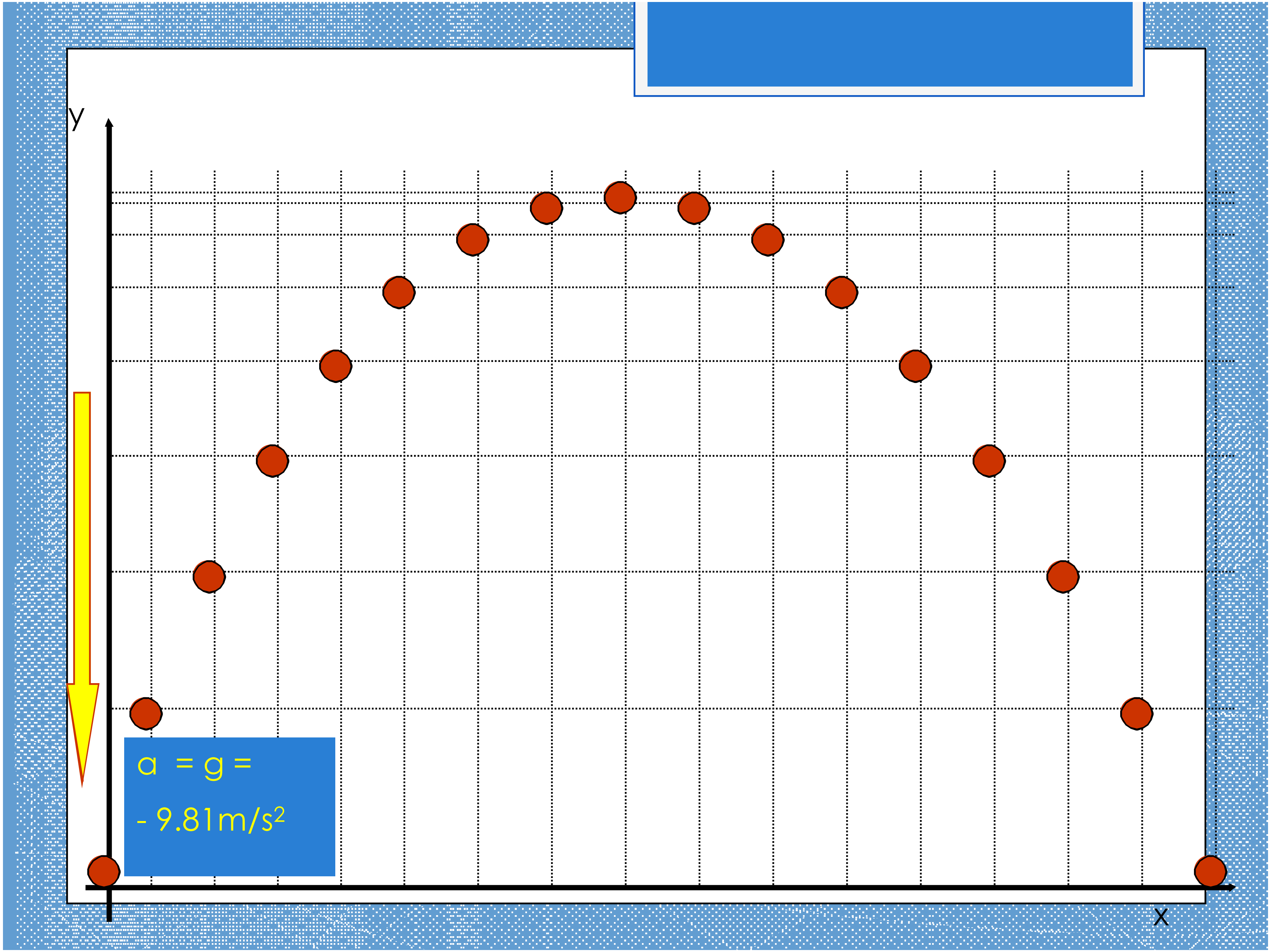


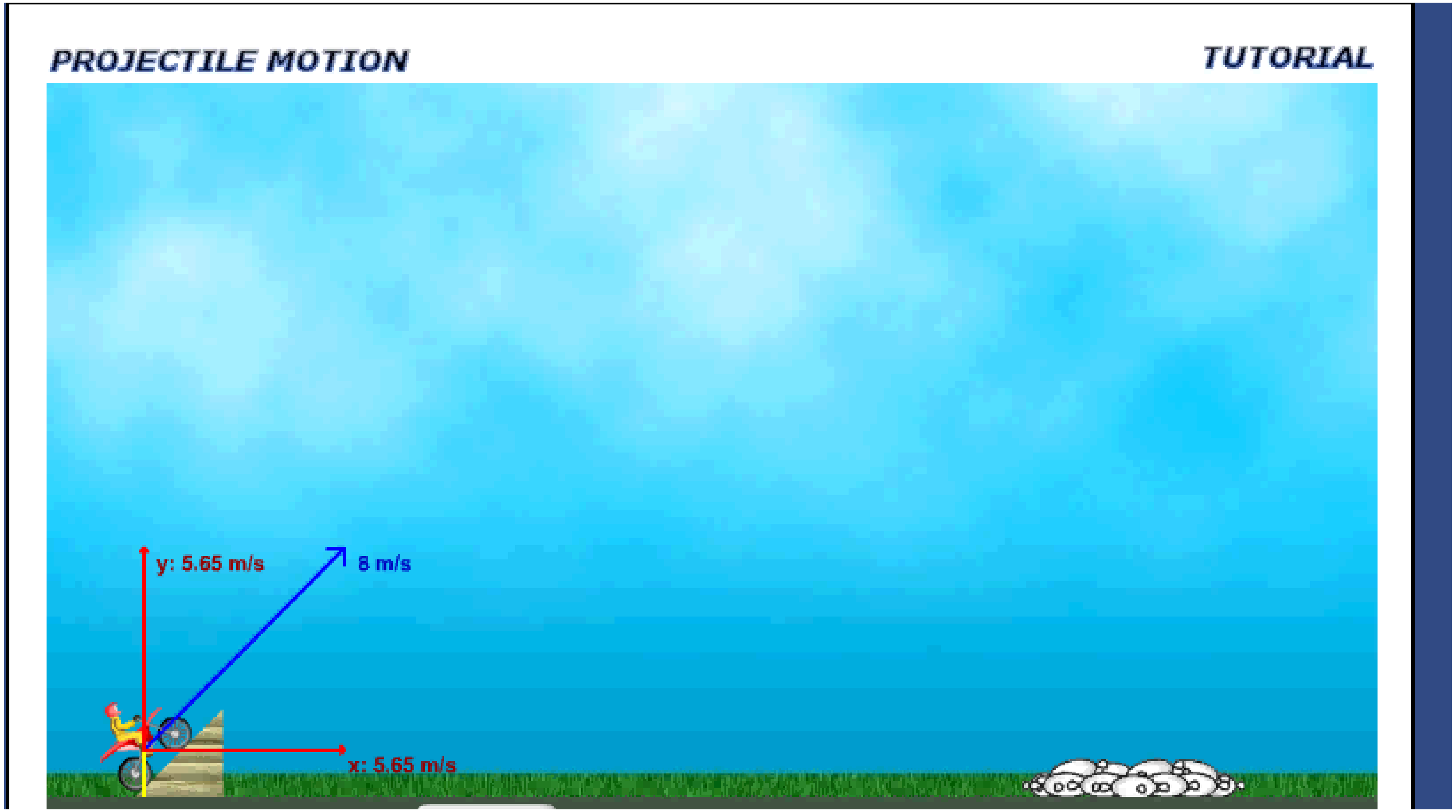
$x = 0, y = 0$

$\vec{v}_i = v_i\,[\Theta]$

$v_{ix} = v_i \cos \Theta$

$v_{iy} = v_i \sin \Theta$


y
x
a = g =
- 9.81m/s2

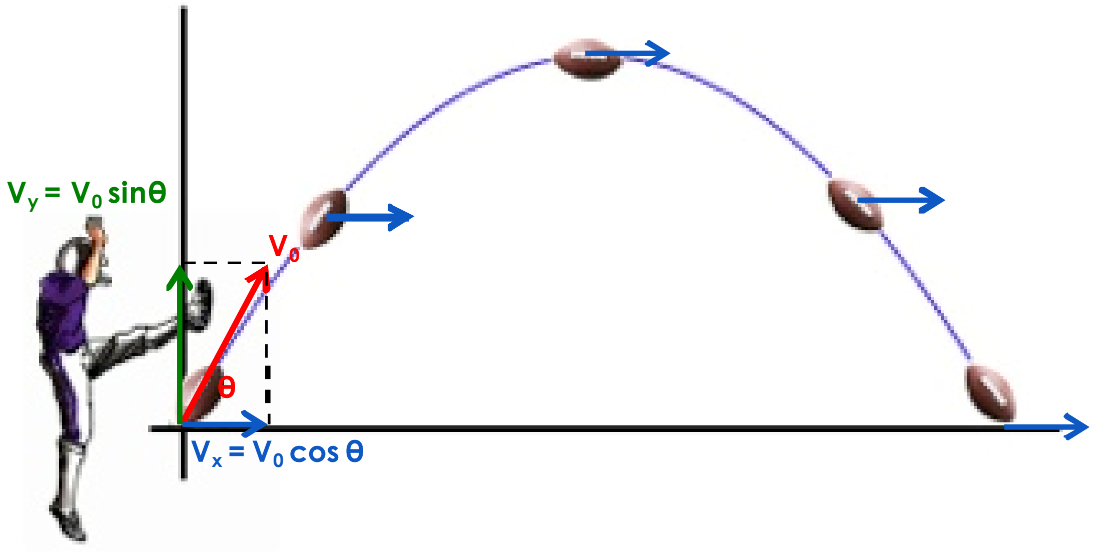

PROJECTILE MOTION
TUTORIAL
y: 5.65 m/s
8 m/s
x: 5.65 m/s

Projectile motion

- Horizontal motion

Horizental motion : Constant velocity, No acceleration

$a_x = 0$
$V_x = V_0 \cos\theta$
$X = V_0 \cos\theta\, t$

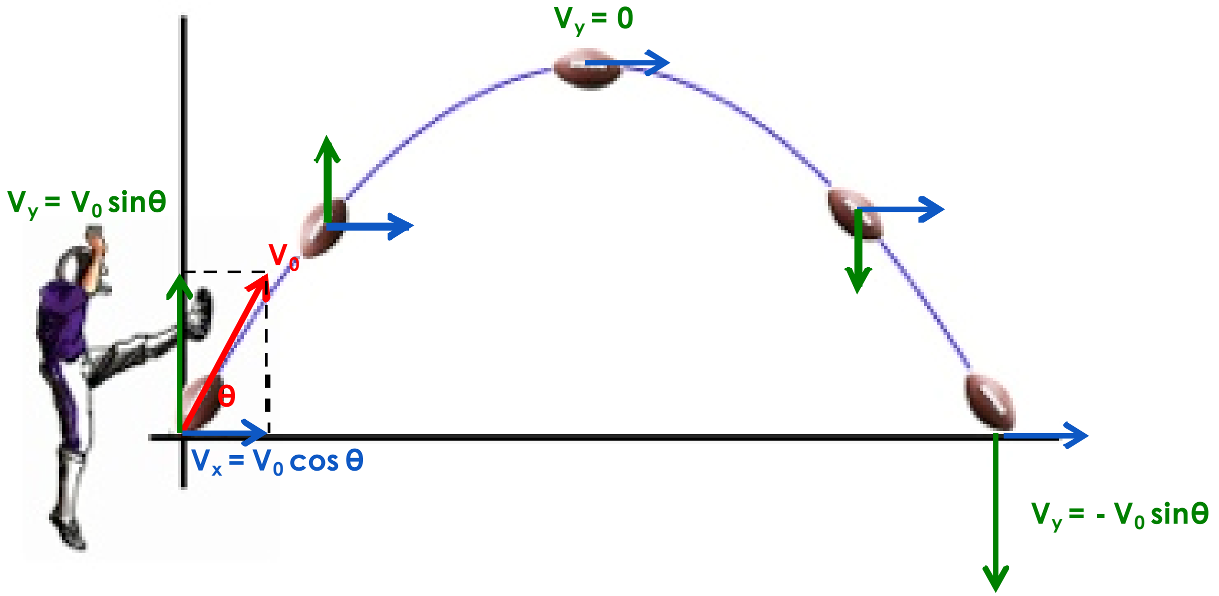


Projectile motion

- Horizontal motion
- Vertical motion

Vertical motion : Constant acceleratin

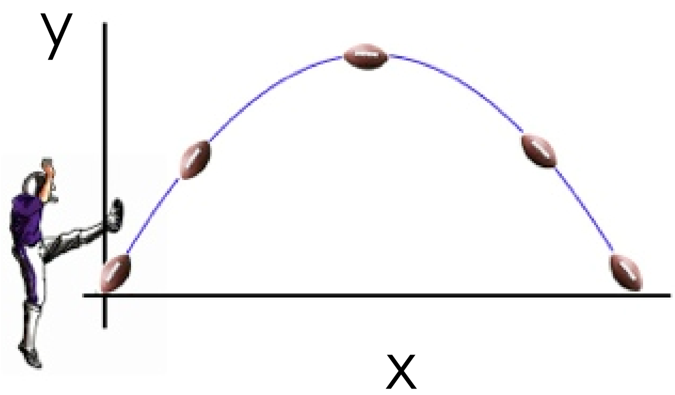
$a_y = g$
$V_y = g\,t + V_0 \sin\theta$
$Y \;= 1/2\, g\, t^2 + V_0 \sin\theta t + Y_0$



	X Uniform motion	Y Accelerated motion
ACCELERATION	$a_x = 0$	$a_y = g = -9.81\ m/s^2$
VELOCITY	$v_x = v_{ix} = v_i \cos \Theta$ $v_x = v_i \cos \Theta$	$v_y = v_{iy} + g\,t$ $v_y = v_i \sin \Theta + g\,t$
DISPLACEMENT	$x = v_{ix}\,t = v_i\,t \cos \Theta$ $x = v_i\,t \cos \Theta$	$y = h + v_{iy}\,t + \frac{1}{2} g\,t^2$ $y = v_i\,t \sin \Theta + \frac{1}{2} g\,t^2$

$x = v_i t \cos \Theta$

$y = v_i t \sin \Theta + ½\, g\, t^2$



$t = x/(v_i \cos \Theta)$

$$y = \frac{v_i x \sin \Theta}{v_i \cos \Theta} + \frac{g x^2}{2 v_i^2 \cos^2 \Theta}$$

$$y = x \tan \Theta + \frac{g}{2 v_i^2 \cos^2 \Theta} x^2$$

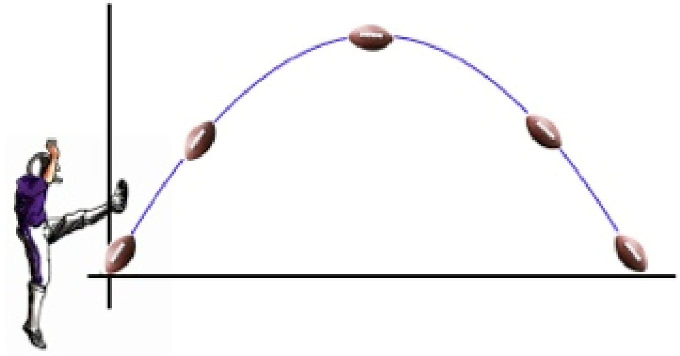
$y = bx + ax^2$

$$y = v_i t \sin \Theta + ½ g t^2$$

$$0 = v_i \Delta t \sin \Theta + ½ g (\Delta t)^2$$

$$0 = v_i \sin \Theta + ½ g \Delta t$$

$$-v_i \sin \Theta = ½ g \Delta t$$



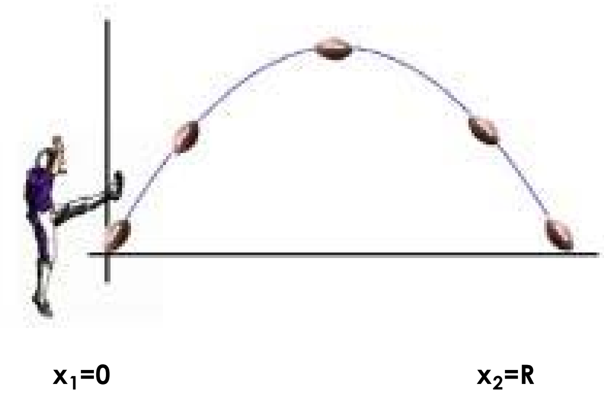
$$\Delta t = \frac{-2 v_i \sin \Theta}{(g)}$$

Horizontal range

$x = v_i t \cos \Theta$

$\Delta x = v_i \Delta t \cos \Theta$

$$\Delta t = \frac{2 v_i \sin \Theta}{(-g)}$$



$\sin (2 \Theta) = 2 \sin \Theta \cos \Theta$

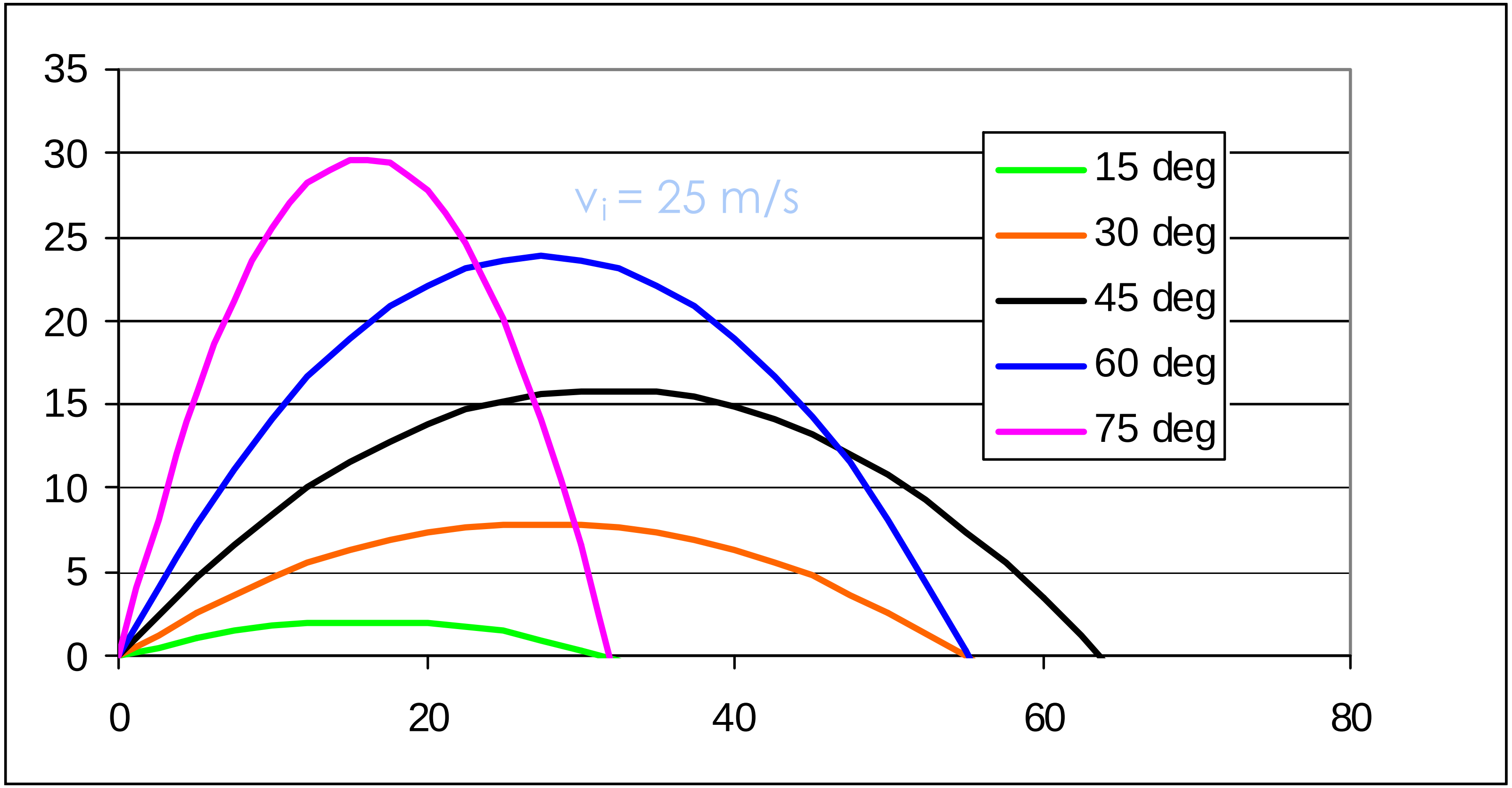
$$\Delta x = \frac{2 v_i^2 \sin \Theta \cos \Theta}{(-g)}$$

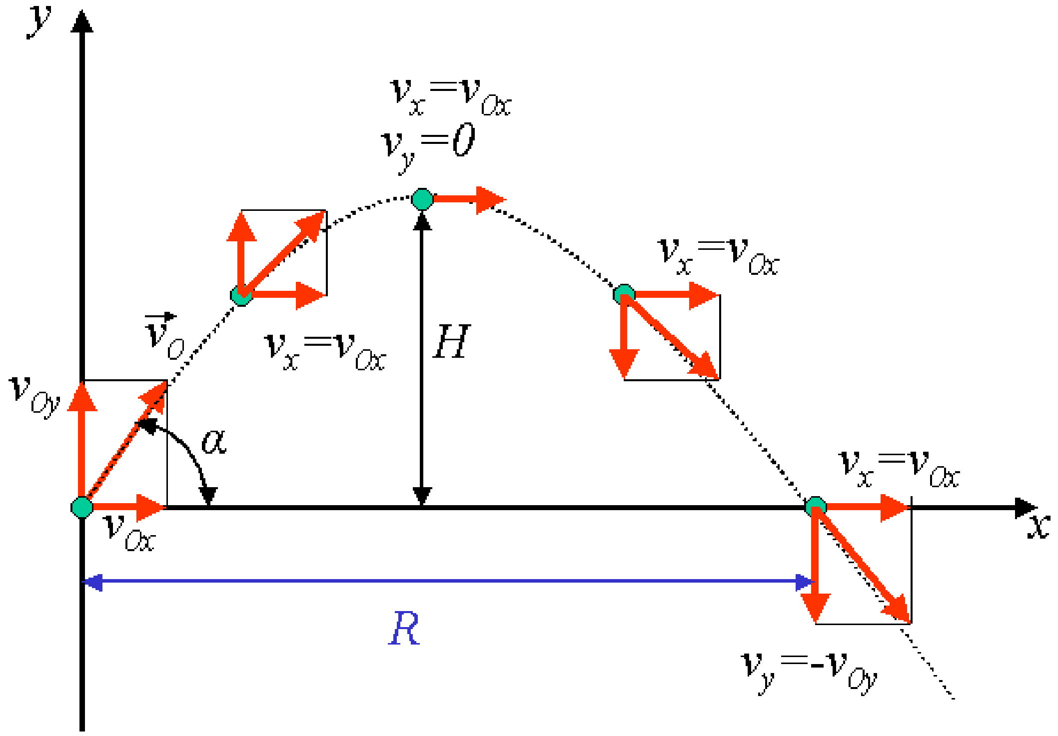
$$\Delta x = \frac{v_i^2 \sin (2 \Theta)}{(-g)}$$

$$\Delta x = \frac{v_i^2 \sin(2\Theta)}{(-g)}$$

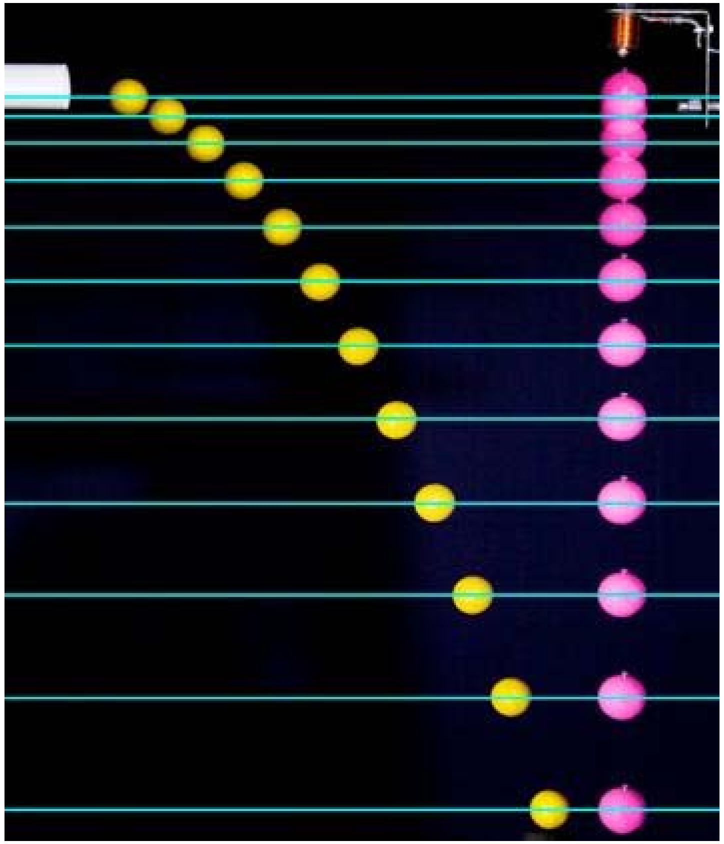
Θ (deg)	sin (2 Θ)
0	0.00
15	0.50
30	0.87
45	1.00
60	0.87
75	0.50
90	0

$$y = x\tan\Theta + \frac{g}{2v_i^2\cos^2\Theta}x^2$$




y
v_x=v_0x
v_y=0
v_x=v_0x
v_0
v_x=v_0x
H
v_0y
α
v_x=v_0x
x
v_0x
R
v_y=-v_0y

The horizontal distance changes as a constant velocity motion.



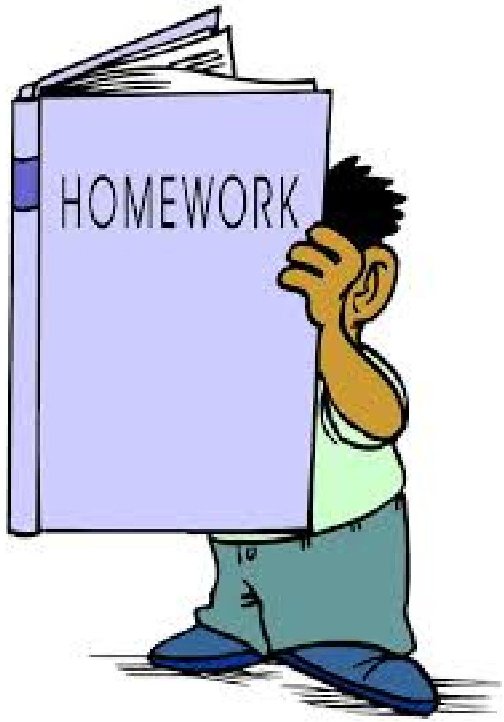
The vertical distance changes as a free fall motion.

Chapter 3

#24 ,#39

Chapter 4

#10 , #18 , #31



Refrences:

- www.worldofteaching.com/powerpoints/physics/projectile%20motion.ppt
- http://cyfair3.schoolwires.net/194820511192159190/lib/194820511192159190/unit_7_projectile_motion.ppt
- http://highered.mcgrawhill.com/sites/dl/free/0073404535/299127/Interactives_ch04_proj.html