Motion in two and three dimensions.
(Part II)

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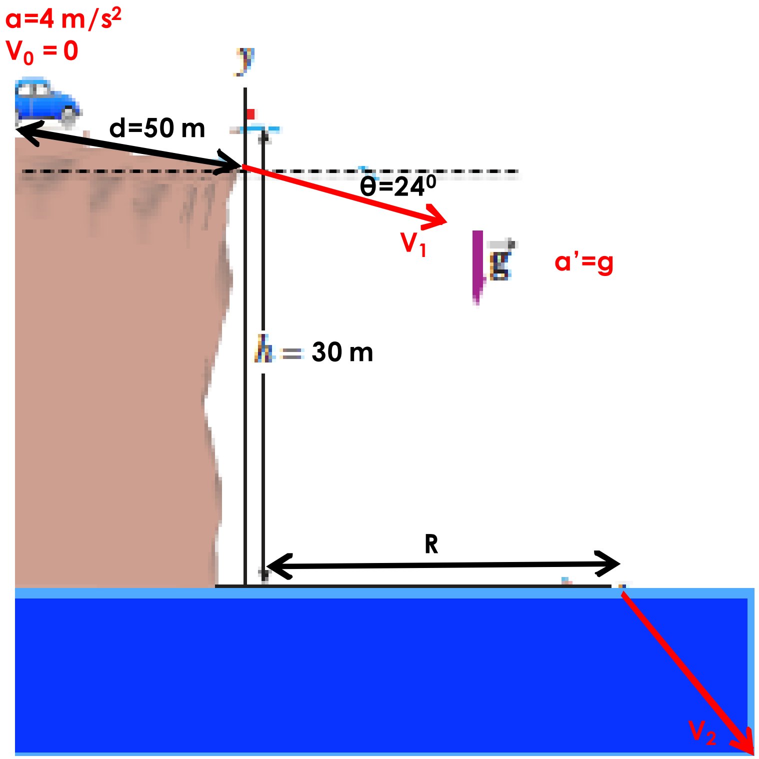
Ex.

A car is parked on a cliff overlooking the ocean on an incline that makes an angle of 24.0° below the horizontal. The negligent driver leaves the car in neutral, and the emergency brakes are defective. The car rolls from rest down the incline with a constant acceleration of 4.00 m/s2 for a distance of 50.0 m to the edge of the cliff, which is 30.0 m above the ocean. Find (a) the car's position relative to the base of the cliff when the car lands in the ocean and (b) the length of time the car is in the air.

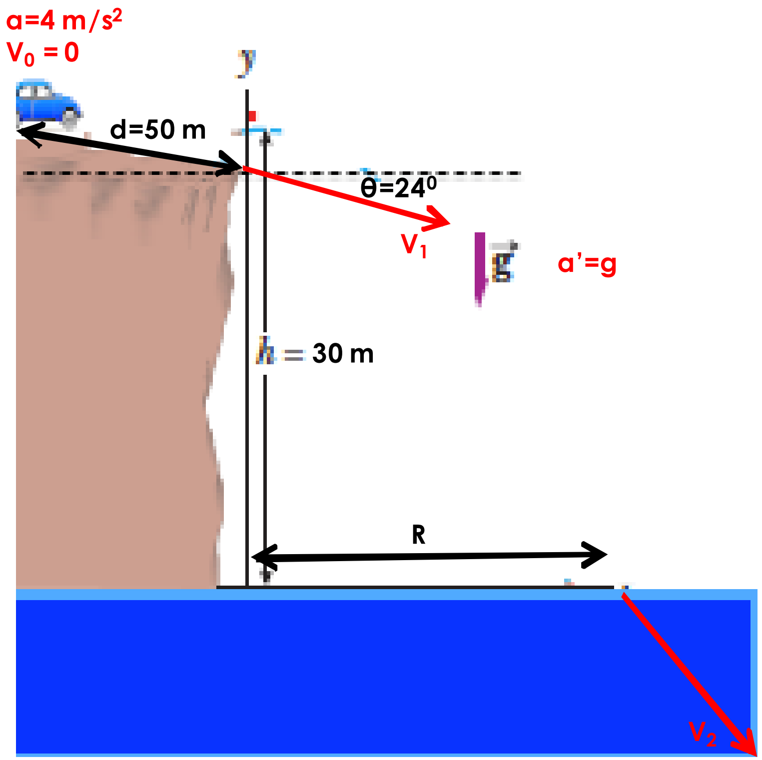


$|V_1|^2 - |V_0|^2 = 2\,a\,d$

$|V_1| = 20$

$V_{1x} = 20 \cos 24^0 \cong$ 18. 27 m/s

$V_{1y} = -20 \sin 24^0 \cong -$ 8. 13 m/s



y: constant acceleration

$V_{2y}^2 - V_{1y}^2 = 2\,a_y\,h$

$V_{2y}^2 - 66.09 = 2\,(9.8)\,30$ $V_{2y} = -25.57$m/s

$V_{2y} = a'\,t + V_{1y} = -9.8\,t - 8.13$

$V_{2y} = -25.57 = -9.8\,T - 8.13$

T = 1.78 s

x : constant velocity

$x = V_{1x}\,t$

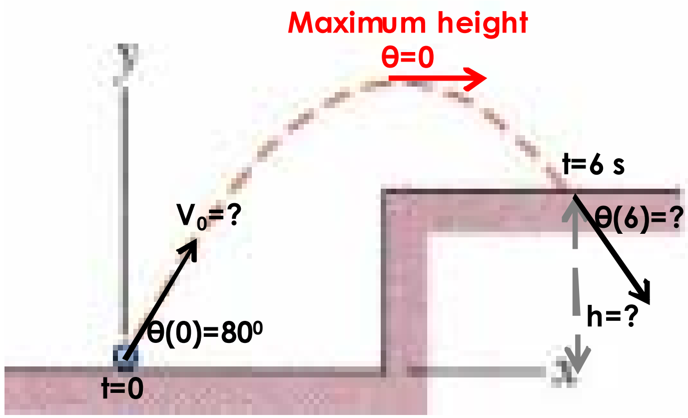
$R = V_{1x}\,T = 18.27 * 1.78$

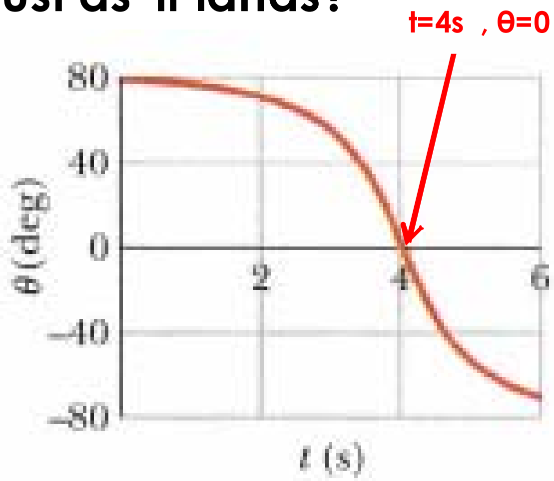
R = 34.34 m

Ex.

At time $t=0$, a golf ball is shot from ground level into the air, as indicated in Fig. The angle θ between the ball's direction of travel and the positive direction of the x axis is given as a function of time.

The ball lands at t : 6.00 s. What is the magnitude v_0 of the ball's launch velocity, at what height $(y - y_0)$ above the launch level does the ball land, and what is the ball's direction of travel just as it lands?





$$V_y(t) = V_0\sin\theta(0) + g\,t$$
$$= V_0\sin 80 \quad + g\,t$$

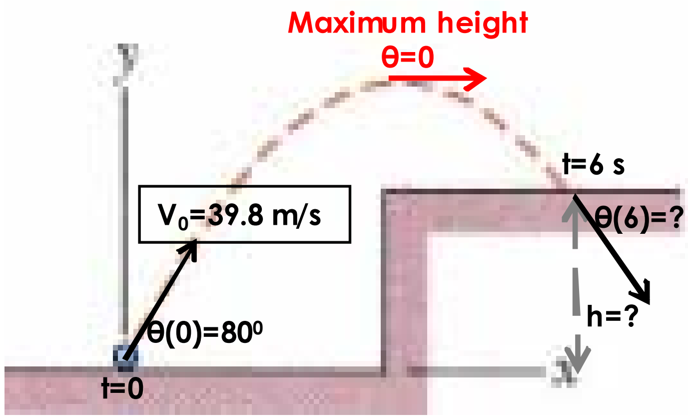
maximum height : $t=4s$, $V_y=0$

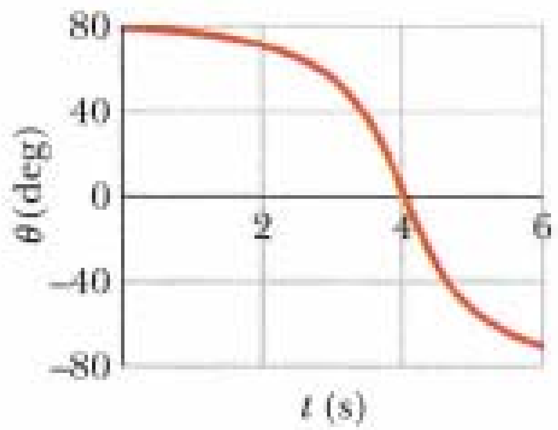
$$0=V_0\sin 80+4g$$

$$\boxed{V_0 = 39.8\ m/s}$$

At time e t - 0, & golf ball is shot from ground level into the air, as indicated in Fig. 4-l8a. The angle θ between the ball's direction of travel and the positive direction of the x axis is given as a function of time.

The ball lands at t : 6.00 s. What is the magnitude v_0 of the ball's launch velocity, at what height $(y – y_0)$ above the launch level does the ball land, and what is the ball’s direction of travel just as it lands?





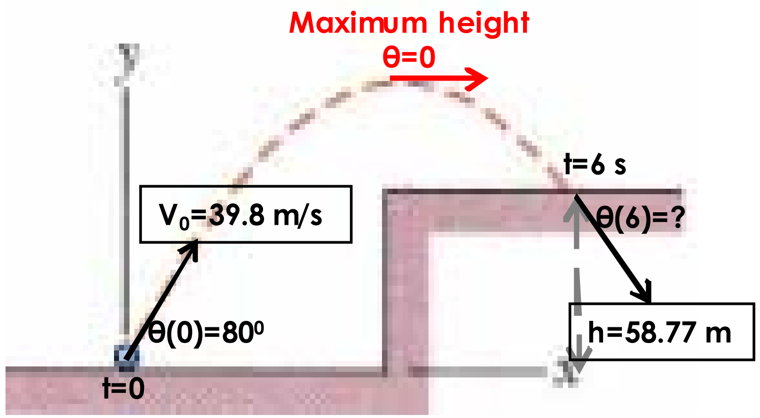
$$y – y_0 = 1/2\ g\ t^2 + V_0 \sin\theta_0 t$$

t= 6s :

$$y(6)– y_0 = h = -1/2\ (9.8)6^2 + 39.8 \sin 80 * 6$$

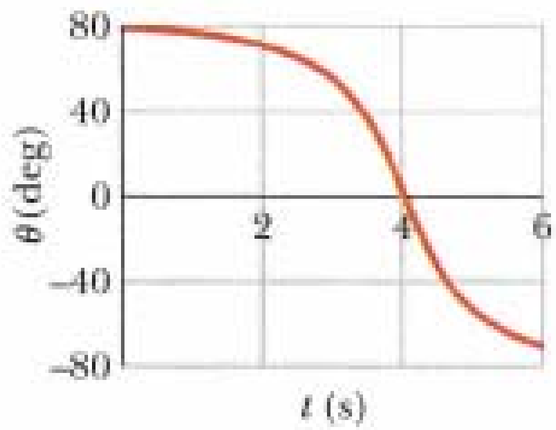
$$\boxed{h=58.77\ m}$$

At time e t - 0, & golf ball is shot from ground level into the air, as indicated in Fig. 4-l8a. The angle θ between the ball's direction of travel and the positive direction of the x axis is given as a function of time. The ball lands at t : 6.00 s. What is the magnitude v_0 of the ball's launch velocity, at what height $(y - y_0)$ above the launch level does the ball land, and what is the ball's direction of travel just as it lands?



$$V_y(t) = V_0 \sin 80 + g\,t$$

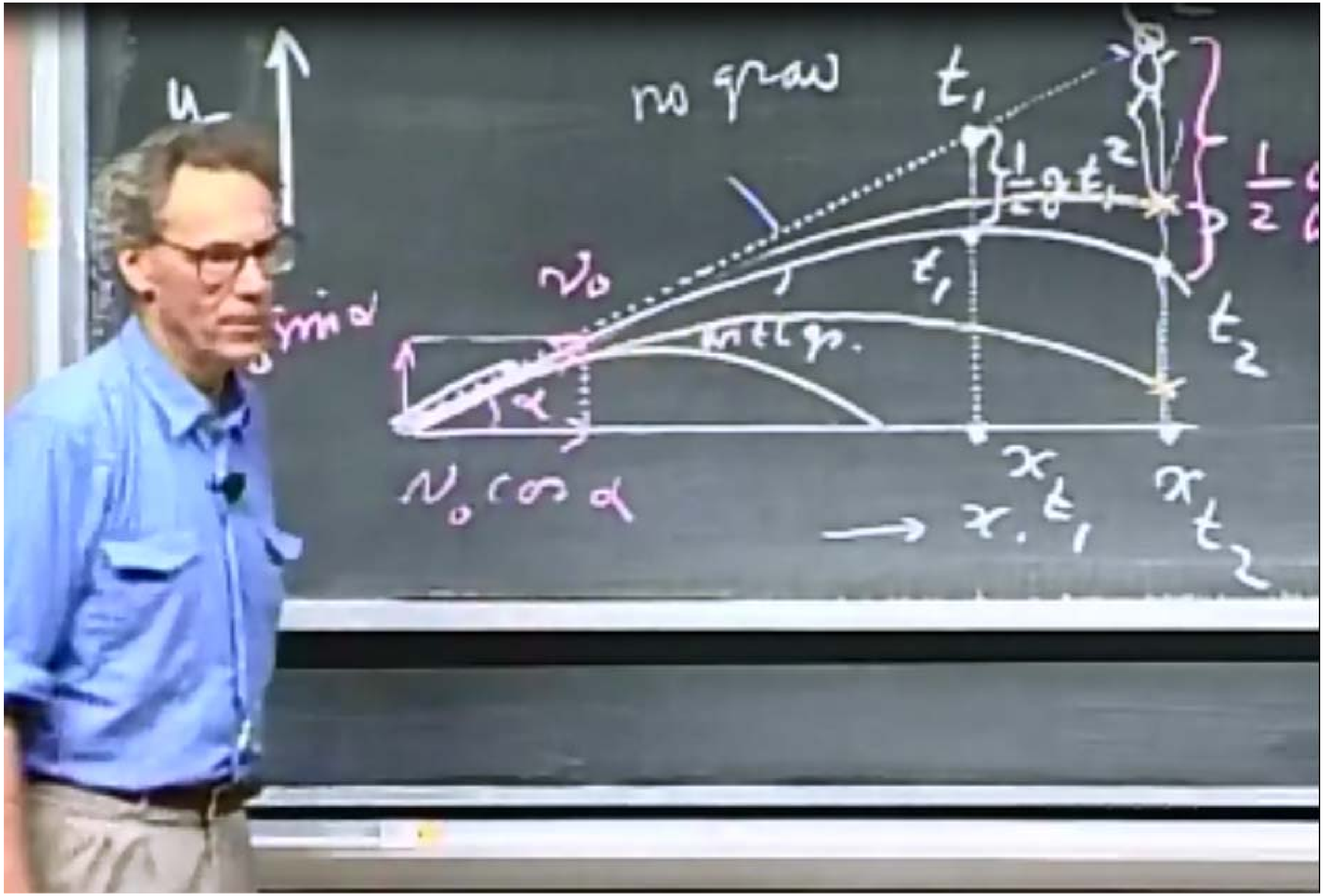
$$V_y(6) = (39.8)\sin 80 + 6g$$
$$= -19.6 \text{ m/s}$$



$$V_x(6) = V_x(0) = V_0 \cos 80$$
$$= (39.8)\cos 80$$
$$= 6.911 \text{ m/s}$$

$$\theta(6) = \tan^{-1}\{V_y(6)/V_x(6)\}$$
$$= \tan^{-1}\{-19.6/6.911\}$$

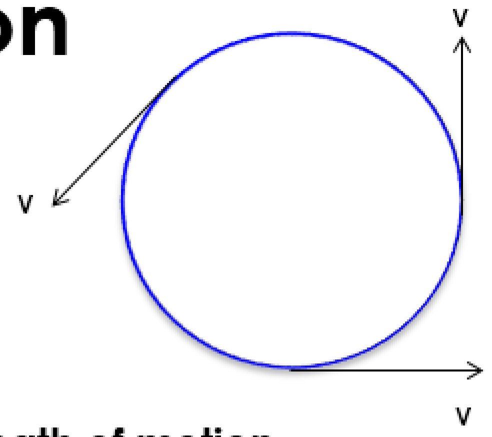
$$\boxed{\theta(6) = -71^0}$$



Lec4 (MIT): 46 Monkey Problem

Uniform Circular Motion

Uniform Circular Motion



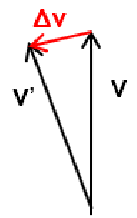
1.Circular path of motion.

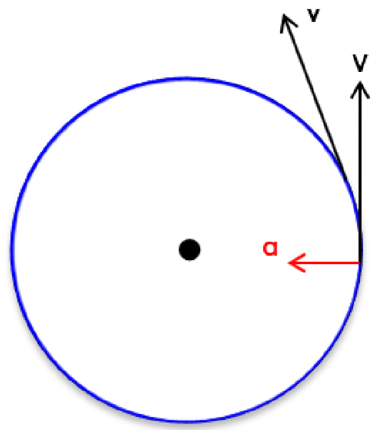
2.The velocity of the particle is tangent to the path of motion.

3. The magnitude of the velocity (speed) is constant.

4. Since the direction of velocity changes, the motion is accelerated.

Uniform Circular Motion



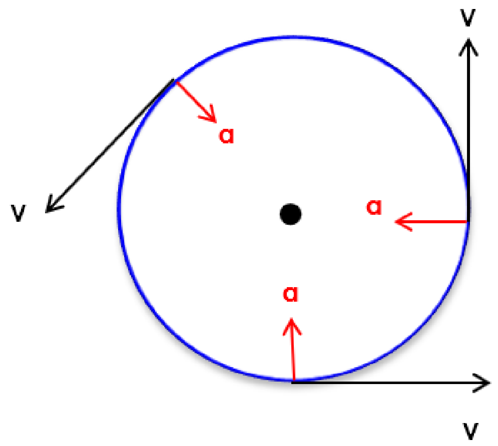


$a = \lim_{\Delta t \to 0} \Delta v/\Delta t = dv/dt$

$\Delta v \; || \; a$

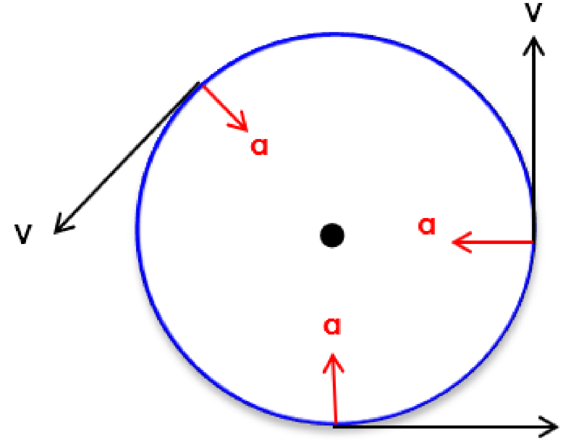
centripetal acceleration

Seeking the center



There is an acceleration due to change in velocity

There must be something that causes this change. Either a *push* or a *pull*.



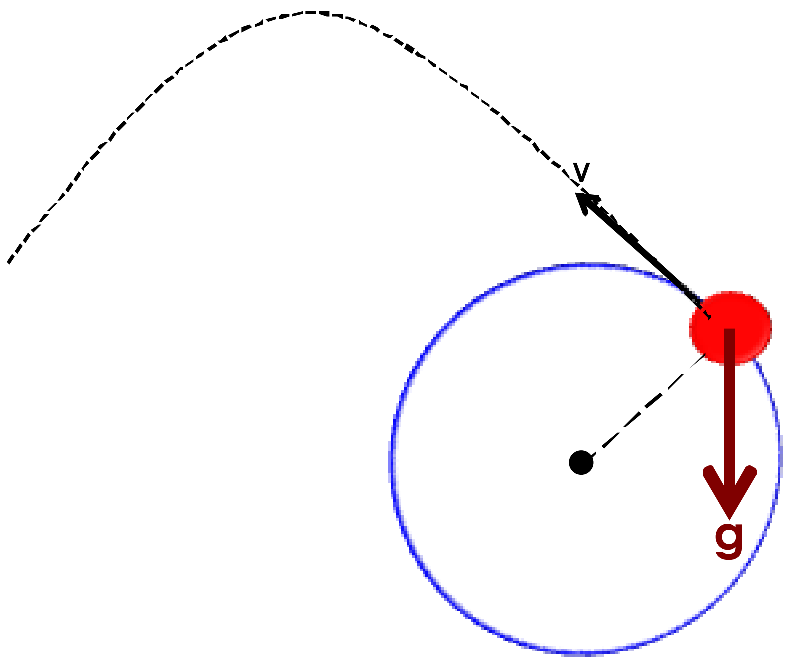
If you are seated on a chair,
you'll feel a ***push*** on your back.

If you are held by a rope,
you'll feel a ***pull*** in your hands.

A ball is connected to the center of a turning table by a rope.

It depends on the moment that we decide to cut the rope.

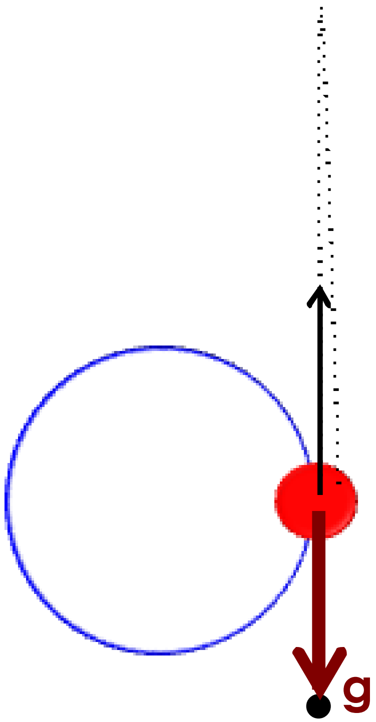
What would be happen if we suddenly cut the rope?



By cutting the rope there would be neither a ***push*** nor a ***pull***. There is only a ball with initial velocity, under the effect of gravitation.

The ball will continue like a projectile.

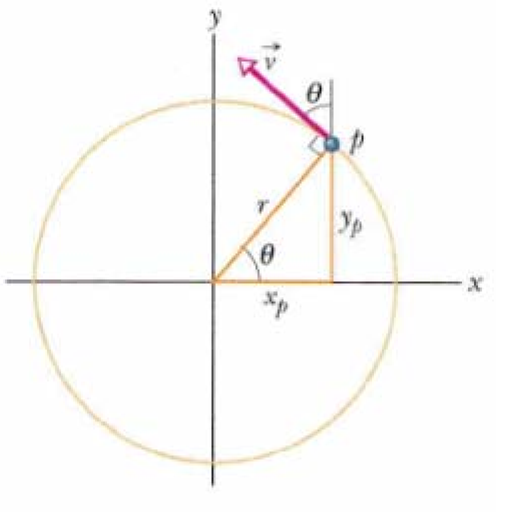
The path of ball will change, if we cut the rope at another moment.



Again,
there would be neither
a ***push*** nor a ***pull***. There is only
a ball with initial velocity,
under the effect of gravitation

The ball will go up and back down.

$\cos\theta = x_p / r$; $\sin\theta = y_p / r$



$V = V_x \hat{\imath} + V_y \hat{\jmath}$

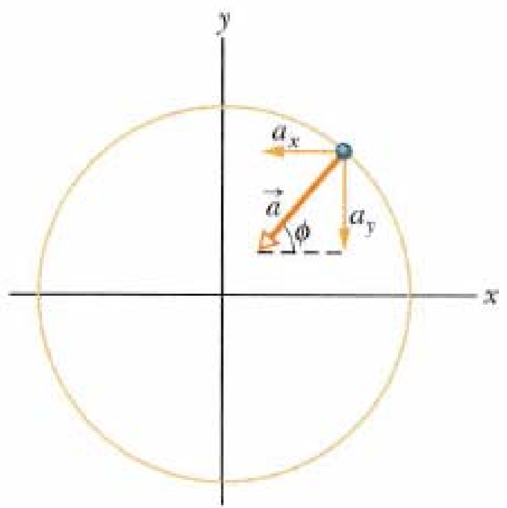
$= -V \sin\theta \hat{\imath} + V \cos\theta \hat{\jmath} = (-V y_p / r)\hat{\imath} + (V x_p / r) \hat{\jmath}$

$a = dV/dt$

$= -V/r \, dy_p/dt \; \hat{\imath} + V/r \, dx_p/dt \; \hat{\jmath}$

$= V/r \; (-V_y \; \hat{\imath} + V_x \; \hat{\jmath})$

$= V/r \; (-V \cos\theta \hat{\imath} - V \sin\theta \; \hat{\jmath})$



$$a = -V^2/r(\cos\theta \; \hat{\imath} + \sin\theta \; \hat{\jmath})$$

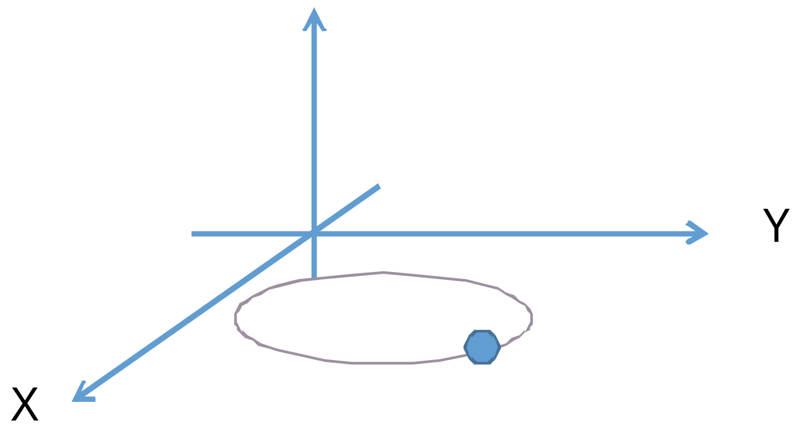
$|a| = V^2/r\sqrt{(\cos^2\theta + \sin^2\theta)}$

$$|a| = V^2/r$$

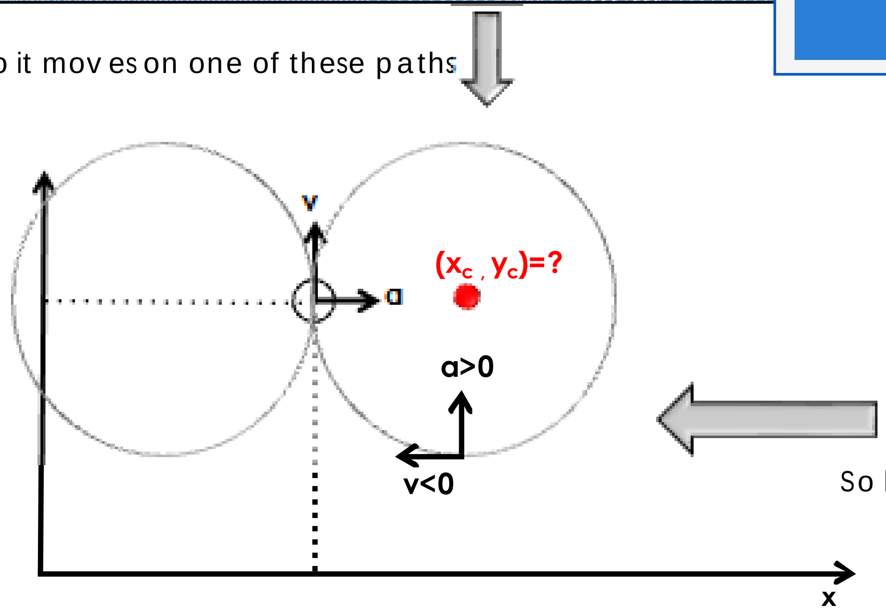
$\tan\varphi = a_y/a_x = \sin\theta/\cos\theta = \tan\theta \longrightarrow$ $\boxed{\varphi = \theta}$

Problem64.

A particle moves along a circular path over a horizantal xy coordinate system, at constant speed. At tima t :4.00 s, it is at point (5.00 m, 6.00 m) with velocity (3.00 m/s) î and acceleration in the positive x direction.
At time t : 10.0 s, it has velocity (-3.00 m/s) î and acceleration in the positive y direction. What are the (a) x and (b) y coordinates of the center of the circular path If $t_2 - t_1$ is less than one period?



So it moves on one of these paths



t: 4s	t: 10s
$x=5$, $y=6$	x , $y=$
$v_x=0$, $v_y=3$	$v_x=-3$, $v_y=0$
$a_x=a$, $a_y=0$	$a_x=0$, $a_y=a$

So his is the only correct path.

$|V| = v$: constant , circular path ➔ uniform circular motion ➔ $|a|=a$

A particle moves along a circular path over a horizantal xy coordinate system, at constant speed.

At time t :4.00 s, it is at point (5.00 m, 6.00 m) with velocity (3.00 m/s) $\hat{j}$ and acceleration in the positive x direction.

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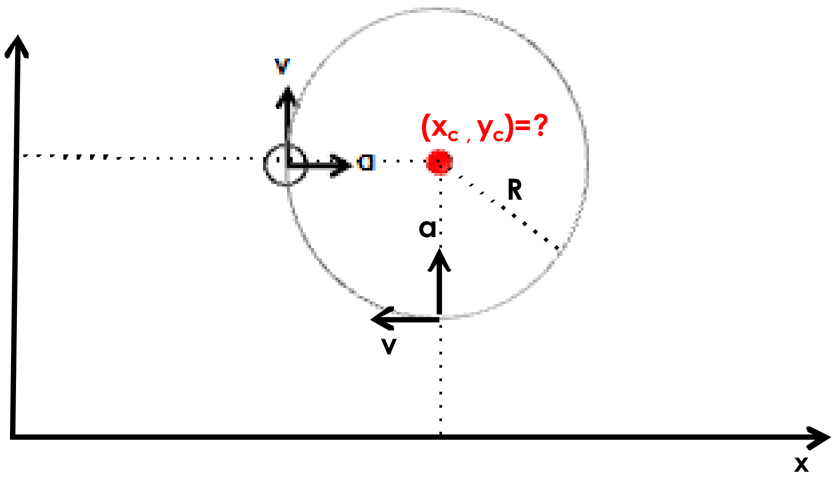


t: 4s	t: 10s
$x=5$, $y=6$	x , $y=$
$v_x=0$, $v_y=3$	$v_x=-3$, $v_y=0$
$a_x=a$, $a_y=0$	$a_x=0$, $a_y=a$

$|V|=R\Delta\theta/\Delta t \xrightarrow{t_2-t_1<2\pi}$

$|V|=3=R(3\pi/2)/(10-4)$
$=R\pi/4$

$\longrightarrow R=12/\pi$

$$(x-x_c)^2+(y-y_c)^2=R^2$$

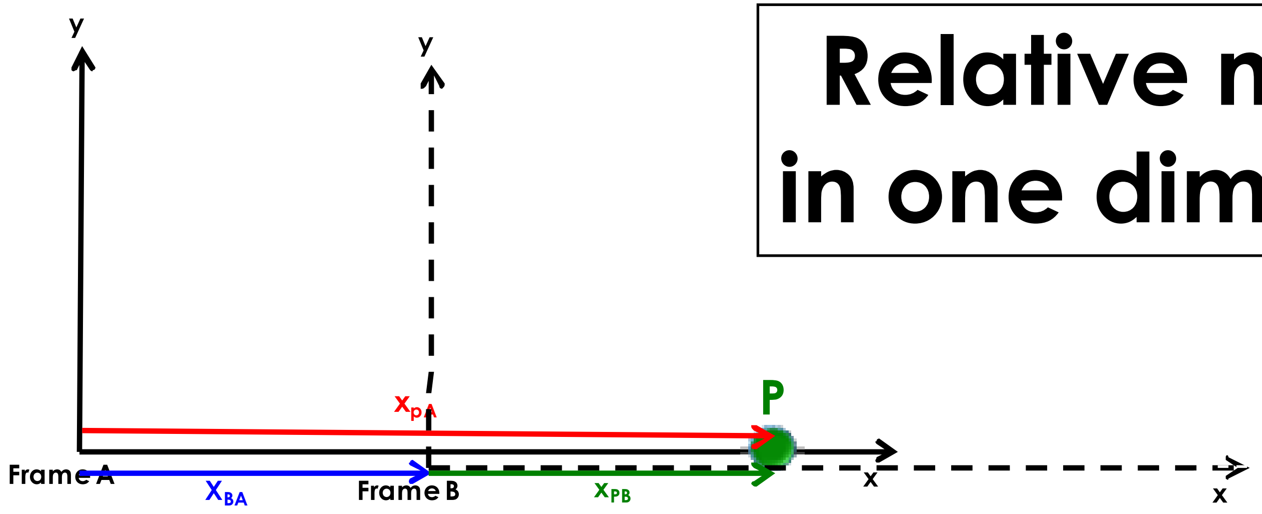
$\xrightarrow{d/dt} (x-x_c)V_x+(y-y_c)V_y=0$

t:4s $(5-x_c)0+(6-y_c)3=0 \longrightarrow y_c=6$

$\xrightarrow{d/dt} v_x^2+(x-x_c)a_x+v_y^2+(y-y_c)a_y=0$

t:4 s $V^2+(5-x_c)a+(6-y_c)0=0 \longrightarrow 5-x_c=-V^2/a=-R \xrightarrow{a=V^2/R} x_c=5+12/\pi$

Relative motion in one dimension

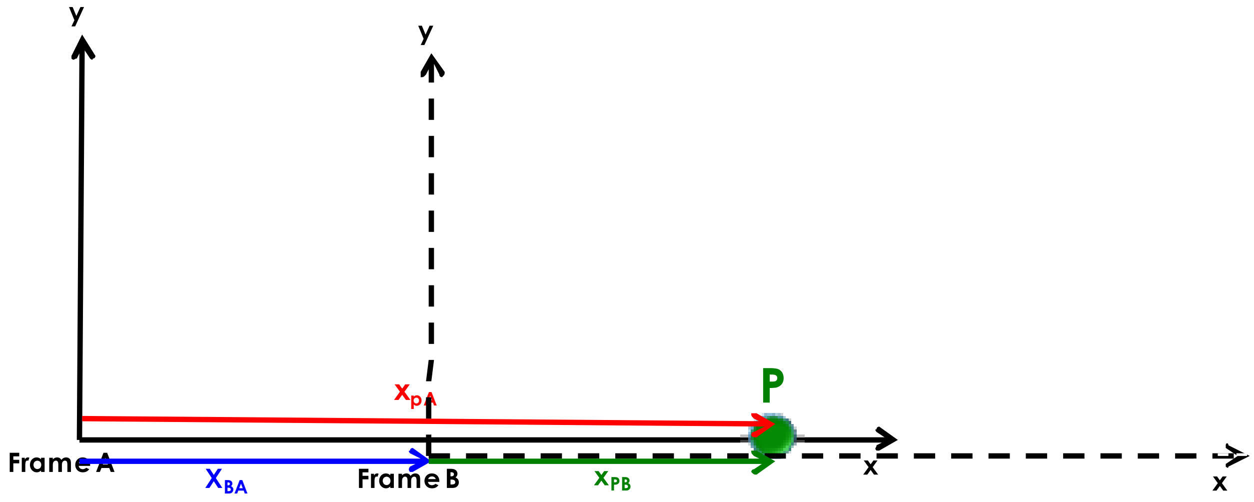


a reference frame Is the physical object to which we attach our cordinate system.

x_{pA} : coordinate of P relative to Frame A

x_{BA} : coordinate of Frame B relative to Frame A

x_{PB} : coordinate of P relative to Frame B



$$x_{PA} = x_{PB} + x_{BA}$$

Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration for a moving particle.

$$\frac{d}{dt}(x_{PA} = x_{PB} + x_{BA})$$

$$v_{PA} = v_{PB} + v_{BA}$$

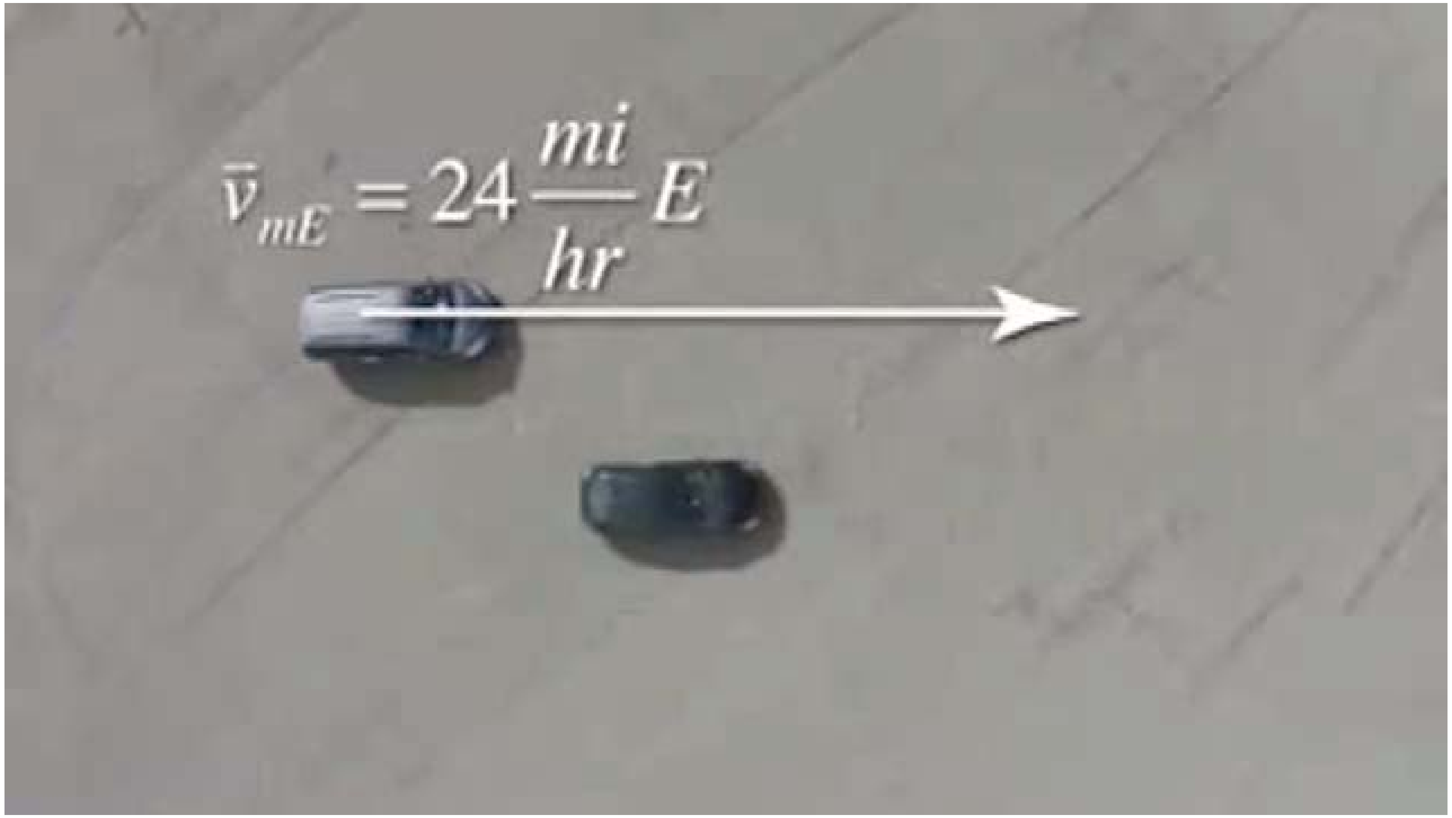
$$\frac{d}{dt}(v_{PA} = v_{PB} + v_{BA})$$

If the two frames move at constant velocity relative to each other ($dV_{PB}/dt=0$)

$$a_{PA} = a_{PB}$$

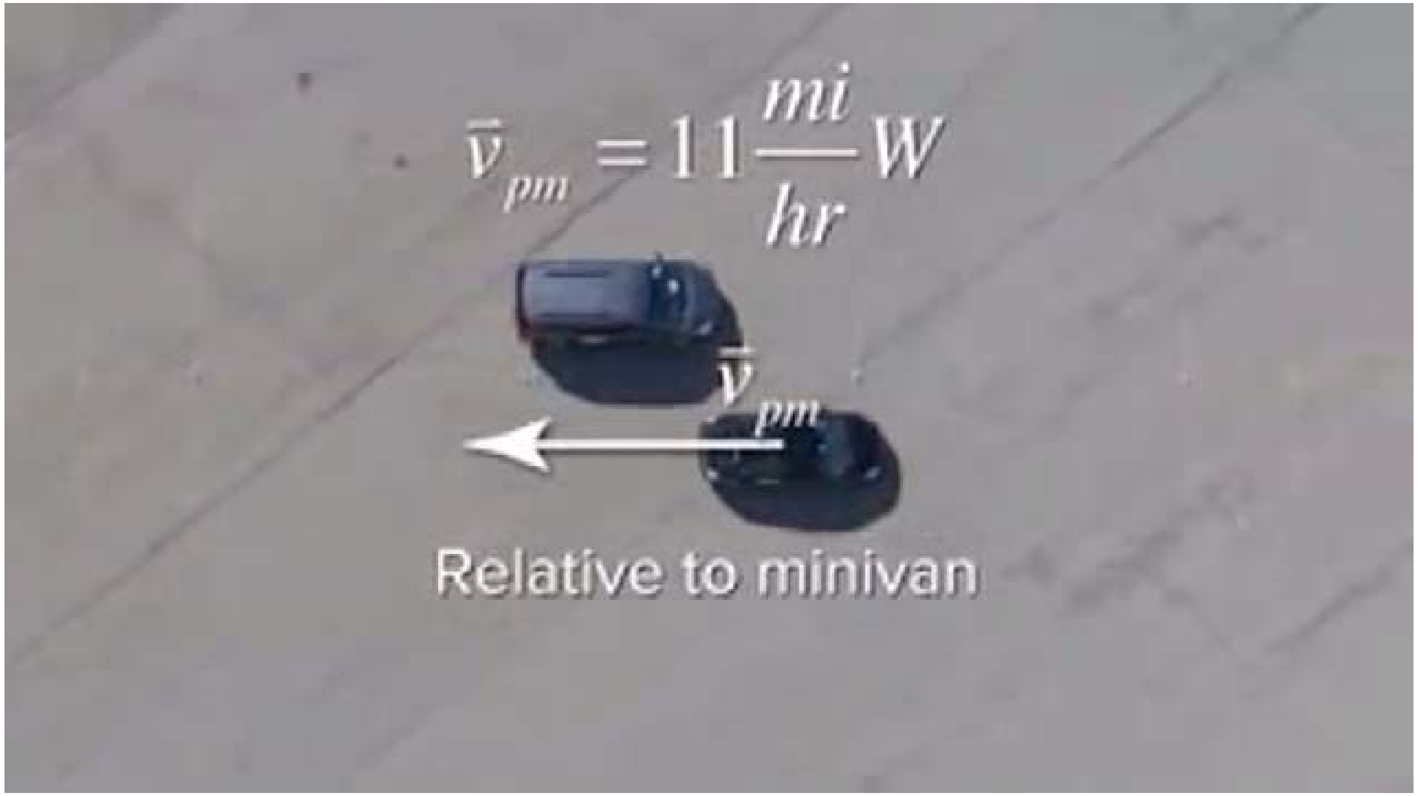
- The camera is attached to the Earth.
- You see the speed of each car relative to the Earth.

- The camera is attached to the Earth
- You see the speed of each car relative to the Earth.

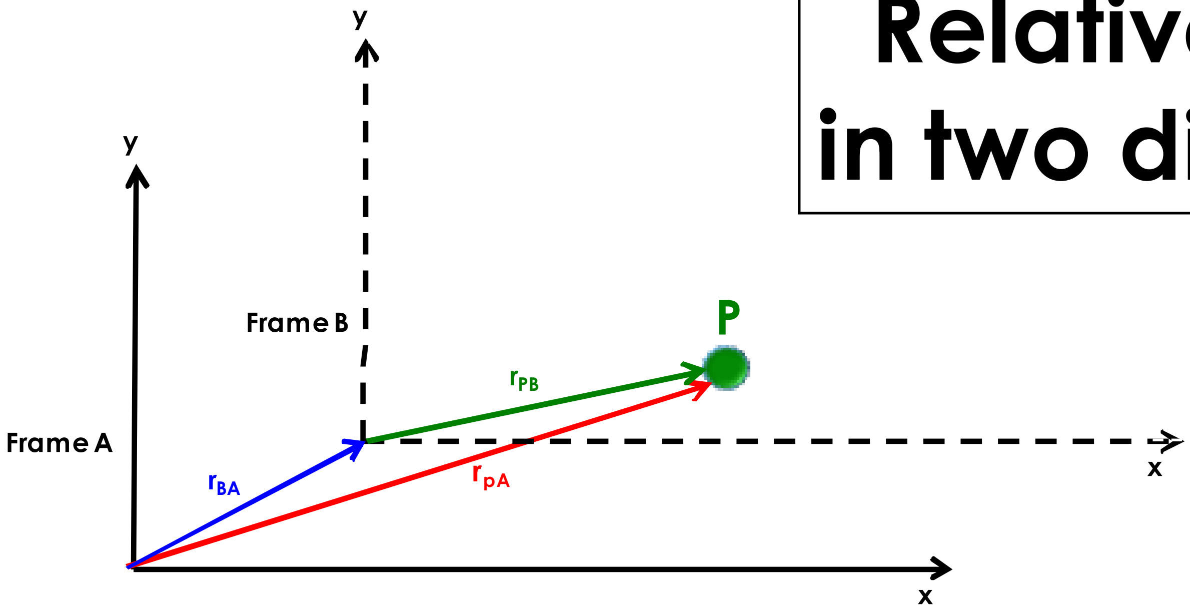


- The camera flies to with prius
- You see the speed of each car relative to Prius.

- The camera flies with minivan.
- You see the speed of each car relative to minivan.



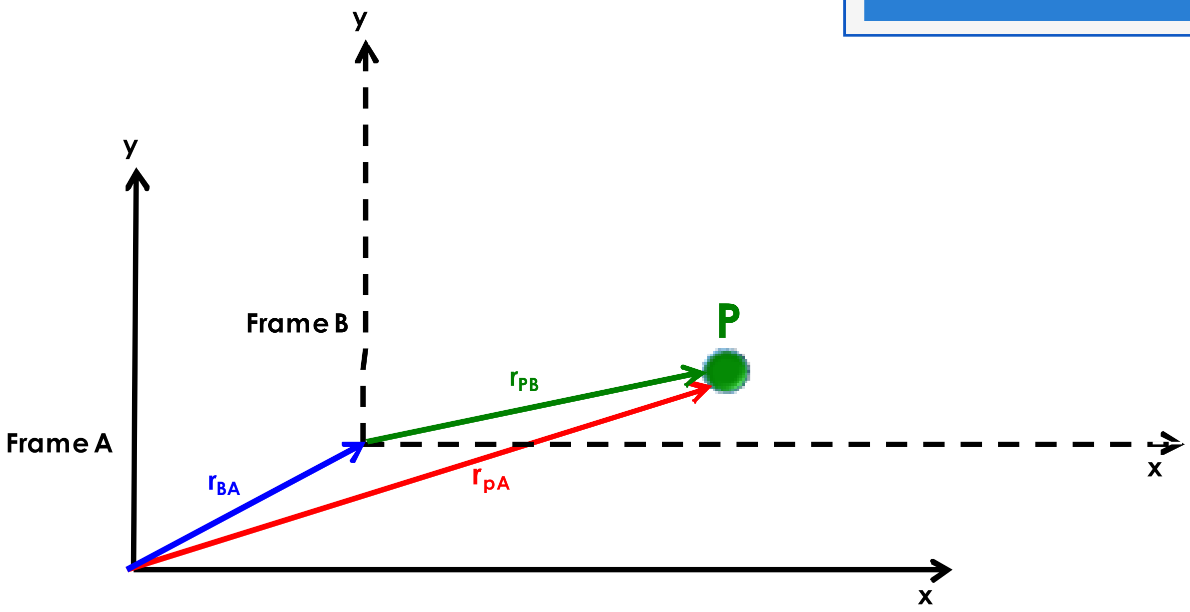
Relative motion in two dimensions



r_{pA} : position vector of P relative to Frame A

r_{BA} : position vector of Frame B relative to Frame A

r_{PB} : position vector of P relative to Frame B



$$r_{PA} = r_{PB} + r_{BA}$$

Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration for a moving particle.

$$\frac{d}{dt}(r_{PA} = r_{PB} + r_{BA})$$

$$V_{PA} = V_{PB} + V_{BA}$$

$$\frac{d}{dt}(V_{PA} = V_{PB} + V_{BA})$$

If the two frames move at constant velocity relative to each other ($dV_{PB}/dt=0$)

$$a_{PA} = a_{PB}$$

Problem71.

A suspicious-looking man runs as fast as he can along a moving sidewalk from one end to the other, taking 2.50 s. Then security agents appear, and the man runs as fast as he can back along the sidewalk to his starting point, taking 10.0 s. What is the ratio of the man's running speed to the sidewalk's speed?

the man runs as fast as he can $\longrightarrow$ $v_{m\text{-}go} = v_{m\text{-}back} = v_m$

the speed of sidewalk is constant $\longrightarrow$ $v_{s\text{-}go} = v_{s\text{-}back} = v_s$

Go: $v_g = d/2.5 = v_m + v_s$

Back: $v_b = -d/10 = -v_m + v_s$

$\longrightarrow$ $v_m = d/4$, $v_s = 3d/20$ $\longrightarrow$ $v_m / v_s = 5/3$