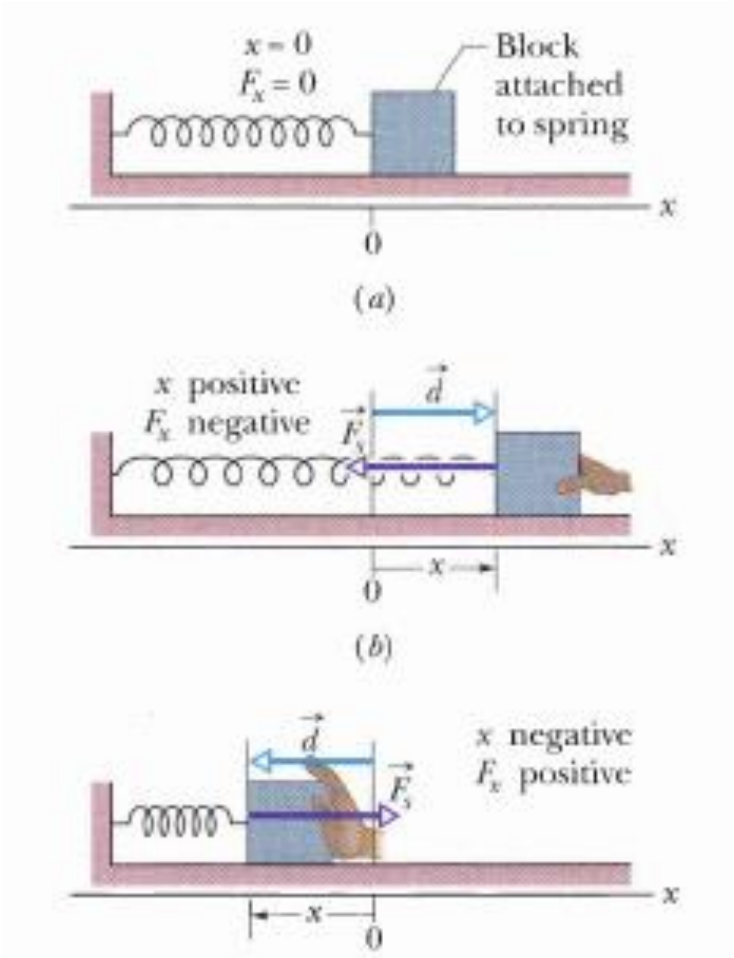
# Hook's law

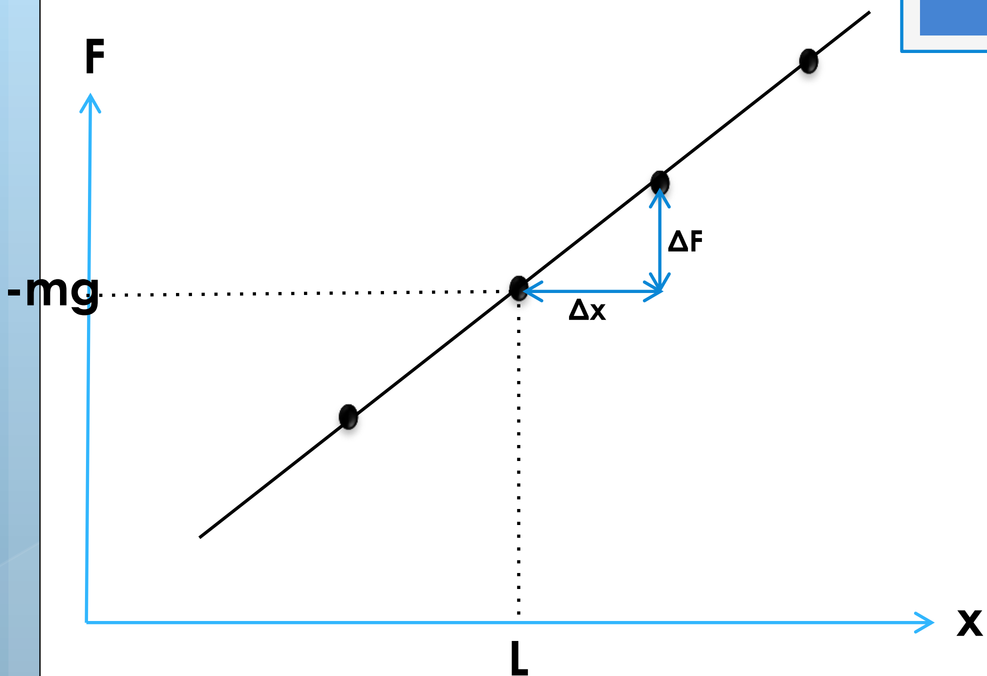



1) F: restoring force

2) |F| ~ |X|

Constant : N/m

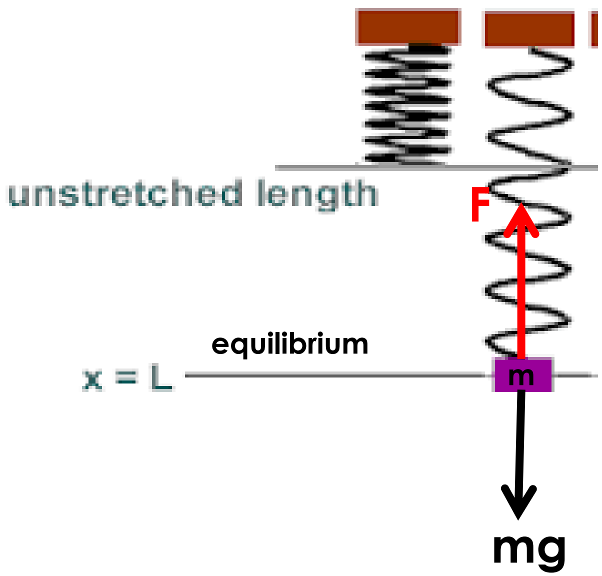
$F = -K x$




We can find different "x" for different "m".

$$K = \Delta F / \Delta x$$

Equilibrium : → $F = -mg$

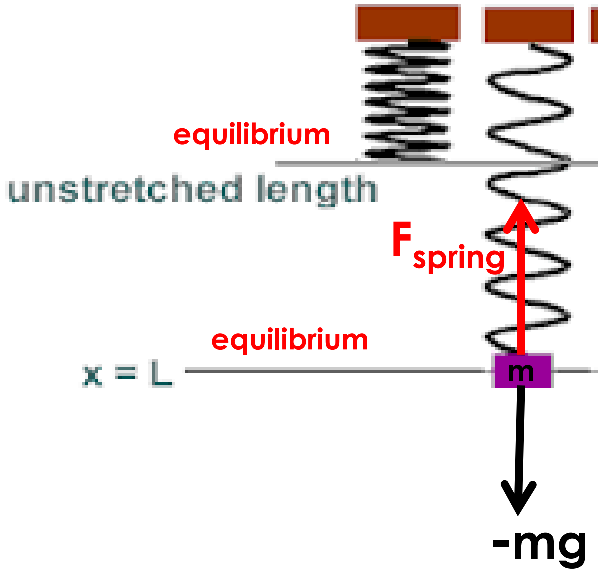



$$W_{AB} = \int_A^B \Sigma F \, dx = \int_A^B -kx \, dx = -k/2 \left(x_B^2 - x_A^2\right)$$

$$\xrightarrow[x_A = 0]{\text{If}} \quad W_{spring} = -\tfrac{1}{2} k x^2$$

## The work done by an applied force:

Equilibrium : → F=-mg




Initial , Final : equilibrium

→ $v_i = v_f = 0$ → $\Delta KE=0$

$\Delta KE = w_{total}$

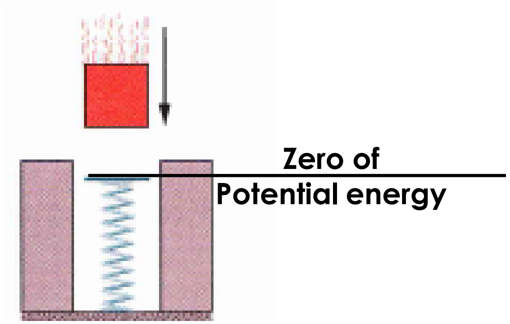
→ $0 = w_{spring} + w_{gravitational}$

→ $w_{gravitational} = -w_{spring}$

→ **If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.**

## Example:

A 250 g block is dropped onto a relaxed vertical spring that has a spring constant of k = 2.5 N/cm (Fig.7-43).The block becomes attached to the spring and compresses the spring 12 cm before momentarily stopping. While the spring is being compressed, what work is done on the block by (a)the gravitational force on it and (b) the spring force? (c) What is the speed of the block just before it hits the spring? (Assume that friction is negligible.) (d) If the speed at impact is doubled, what is the maximum compression of the spring?




$w_g$ = mgy =0.25*-9.8*-0.12= 0.294 j

$w_s = -\frac{1}{2} k y^2 = -1.8$ j

$\Delta KE = w_{total}$ → $\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = w_g + w_s$ → $-0.125\, v_i^2 = -1.506$

$v_i = \sqrt{12}$

↓

$-\frac{1}{2} m v_i'^2 = mgy' - \frac{1}{2} k y'^2$ , $v'_I = 2v_i = 2\sqrt{12}$

↓ $y'^2 + 0.02\, y - 0.048 = 0$ → $y' = -0.229$ m

# Power:

$$P_{avg} = \frac{W}{\Delta t} \quad \text{(average power).}$$

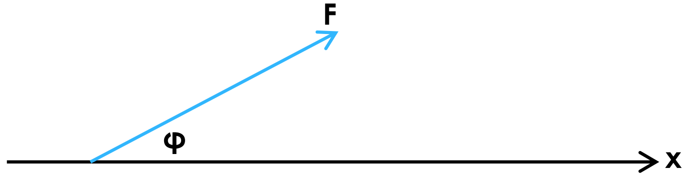
$$P = \frac{dW}{dt} \quad \text{(instantaneous power).}$$

**P : watt: w: j/s**

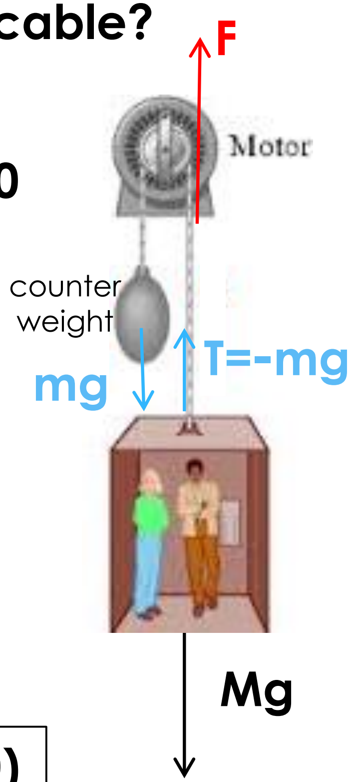
$$P = \frac{dW}{dt} = \frac{F\cos\phi\, dx}{dt} = F\cos\phi\left(\frac{dx}{dt}\right),$$

$$P = Fv\cos\phi.$$

$$P = \vec{F}\cdot\vec{v} \quad \text{(instantaneous power).}$$



A fully loaded, slow-moving freight elevator has a cab with a total mass of 1200 kg, which is required to travel upward 54 m in 3.0 min, ***starting and ending at rest***. The elevator's counterweight has a mass of only 950 kg, and so the elevator motor must help. What average power is required of the force the motor exerts on the cab via the cable?




***Starting and ending at rest*** → $\Delta KE = 0$ → $W_{total}=0$

$$w_{cabin}+w_{motor}+w_{counterweight}=0$$

$$Mgd+Fd-mgd=0 \longrightarrow F=9.8(M-m)$$

$$\longrightarrow P_{motor}= 9.8(M-m)*d/T$$

$$\longrightarrow P_{motor}=(10)(54)(1200-950)/(3*60)$$
$$=750 \text{ w}$$

# A variable force:

$$W = \int_{x_i}^{x_f} F(x)\, dx = \int_{x_i}^{x_f} ma\, dx,$$

$$ma\, dx = m \frac{dv}{dt} dx.$$

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx} v,$$

$$ma\, dx = m \frac{dv}{dx} v\, dx = mv\, dv.$$

$$W = \int_{v_i}^{v_f} mv\, dv = m \int_{v_i}^{v_f} v\, dv = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

**Work-kinetic energy theorem**
**For a variable force**

**A single force acts on a 3.0 kg particle like object whose position is given by $x = 3t - 4t^2 + t^3$, with x in meters and t in seconds. Find the work done on the object by the force from t : 0 to t :4.0 s.**

$$W = \int_{v_i}^{v_f} mv\, dv = m \int_{v_i}^{v_f} v\, dv$$
$$= \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2.$$

**$v = dx/dt = 3 - 8t + 3t^2$ → $v(0) = 3$ m/s , $v(4) = 19$ m/s**

**$w = \frac{1}{2}(3)(19^2 - 3^2) = 528$ j**

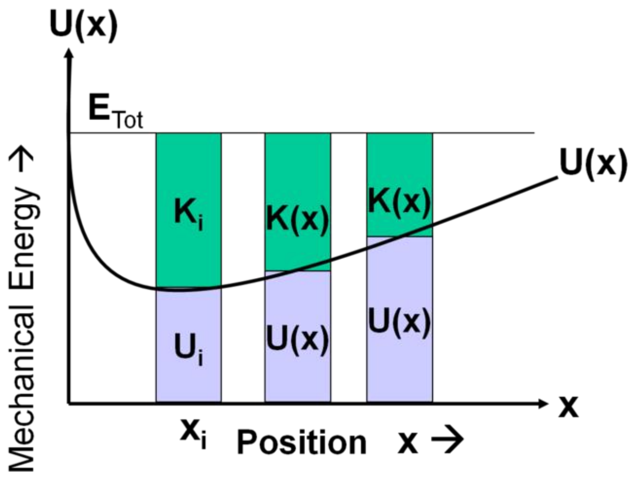
# Reading a Potential Energy Curve

$$\Delta U(x) = -W = -F(x)\,\Delta x.$$

$$F(x) = -\frac{dU(x)}{dx} \qquad \text{(one-dimensional motion),}$$

**For a spring :** $u = \frac{1}{2} kx^2$ $\quad -du/dx = -kx = F_{spring}$ ✓

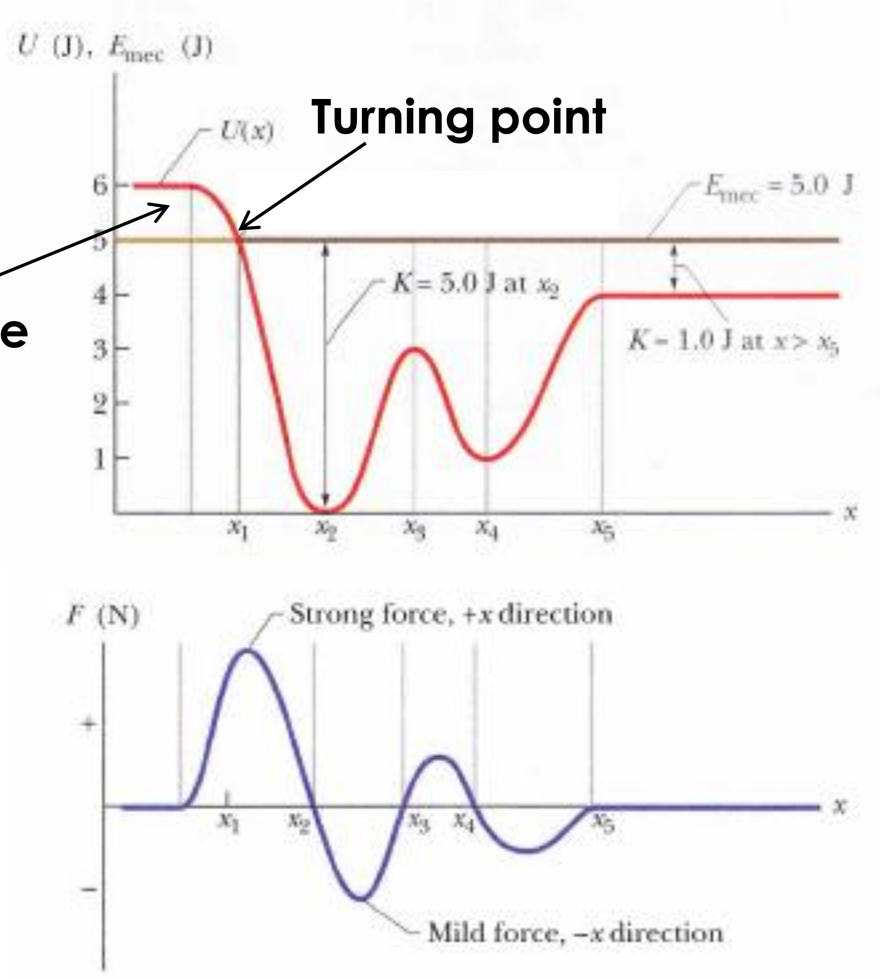
**For a mass :** $u = -mgx$ $\quad -du/dx = mg = F_{gravitaional}$ ✓



k <0 :impossible

-du/dx =F>0

Turning point



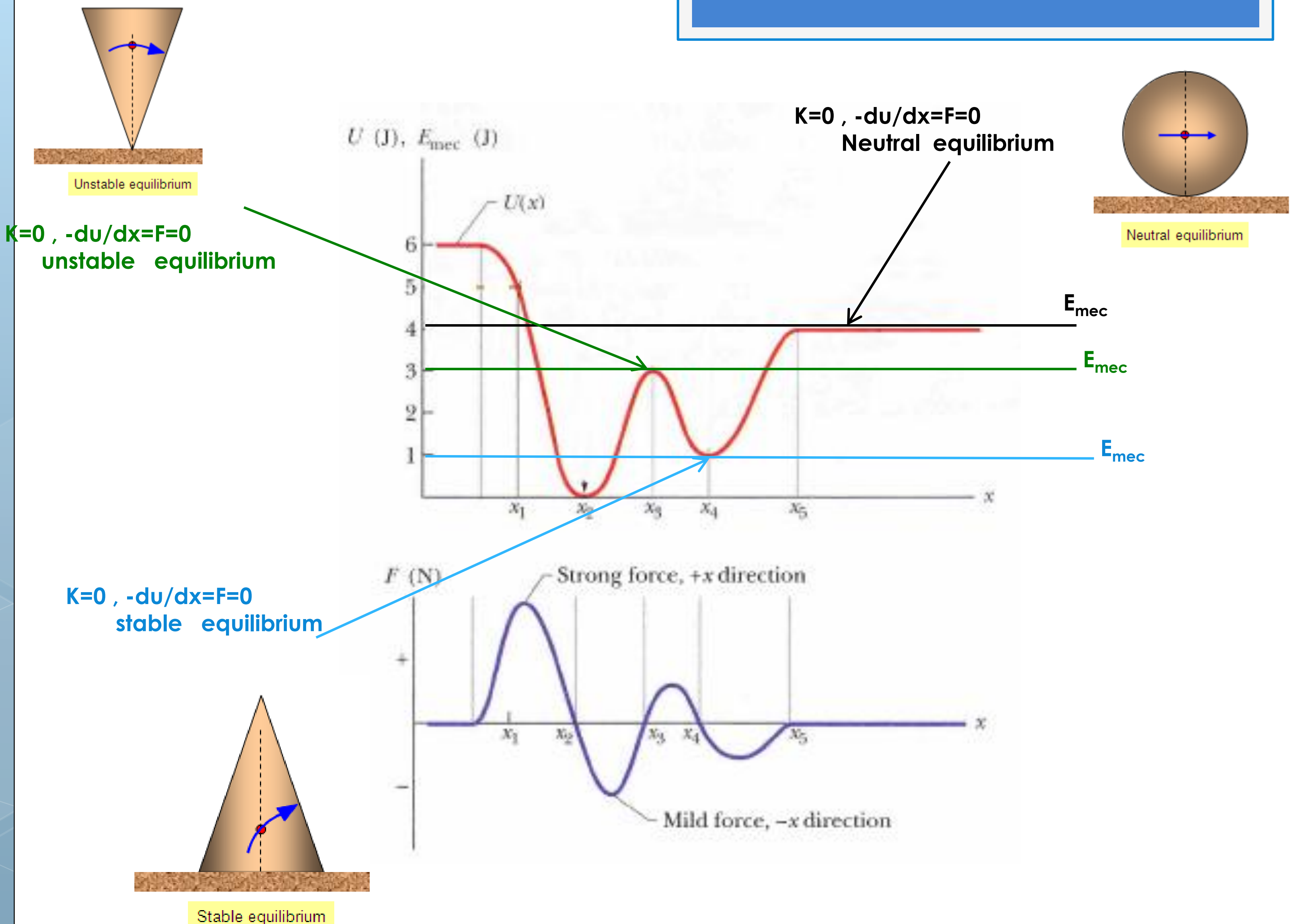
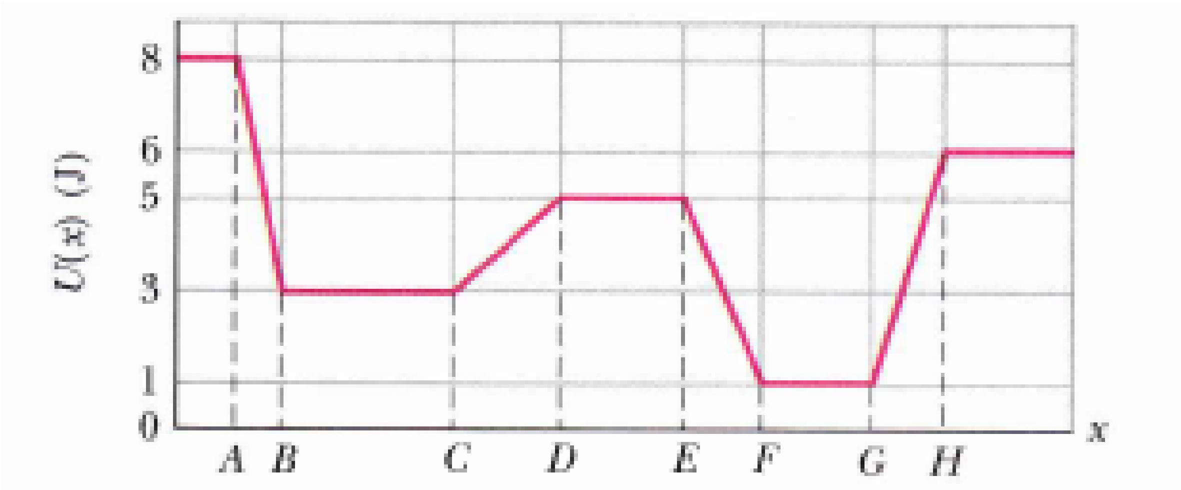
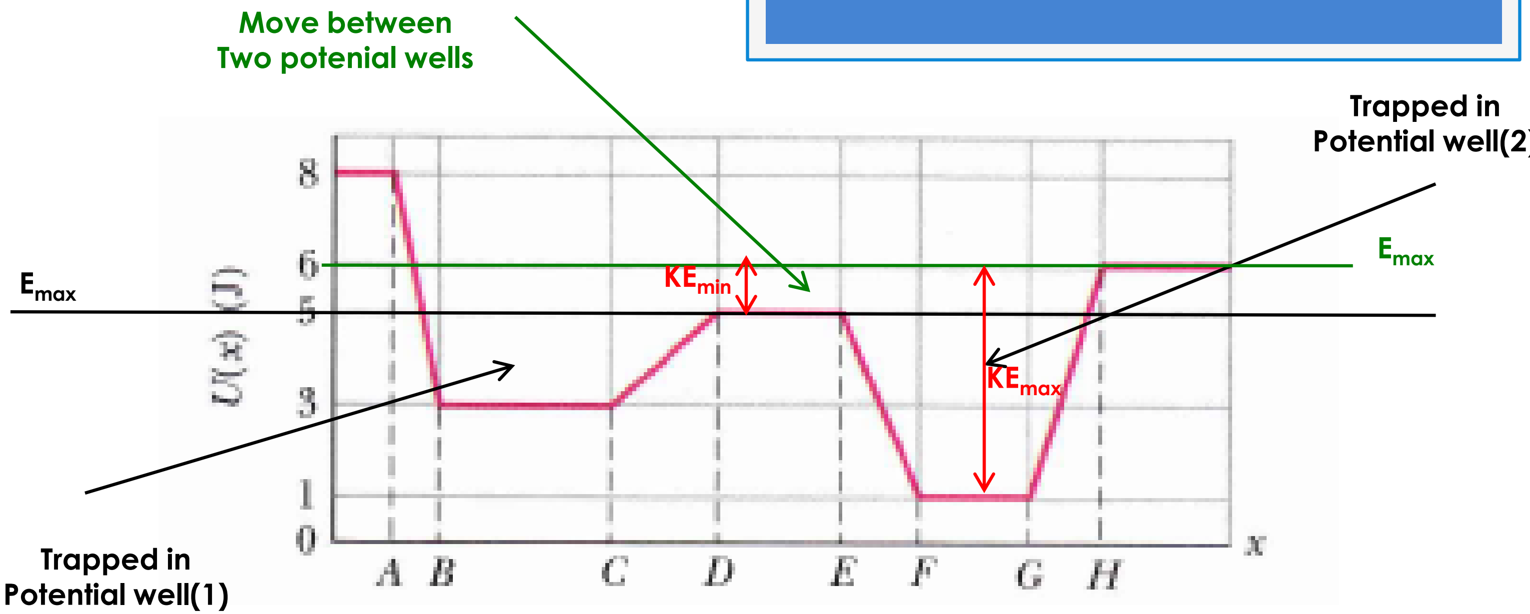

Unstable equilibrium
K=0 , -du/dx=F=0
unstable equilibrium
K=0 , -du/dx=F=0
Neutral equilibrium
Neutral equilibrium
U (J), E mec (J)
U(x)
6
5
4
3
2
1
x
x1
x2
x3
x4
x5
E mec
E mec
E mec
F (N)
Strong force, +x direction
+
–
x
x1
x2
x3
x4
x5
Mild force, –x direction
K=0 , -du/dx=F=0
stable equilibrium
Stable equilibrium

Figure below gives the potential energy function of a particle.
(a) Rank regions AB, BC, CD, and DE according to the magnitude of the force on the particle, greatest first. What value must the mechanical energy $E_{mec}$ of the particle not exceed if the particle is to be (b) trapped in the potential well at the left, (c) trapped in the potential well at the right, and (d) able to move between the two potential wells but not to the right of point H? For the situation of (d), in which of regions BC, DE,and FG will the particle have (e) the greatest kinetic energy and (f ) the least speed?






$$F(x) = -\frac{dU(x)}{dx} \quad \text{(one-dimensional motion)},$$

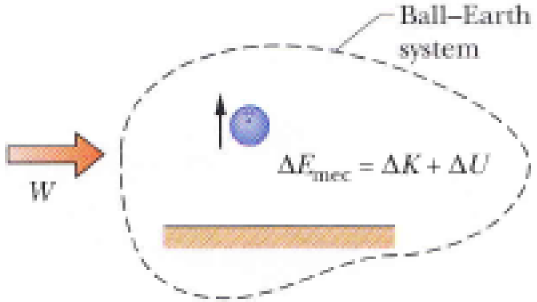
$$|F_{AB}| > |F_{CD}| > |F_{BC}| = |F_{DE}| = 0$$

# The work done on a system of objects

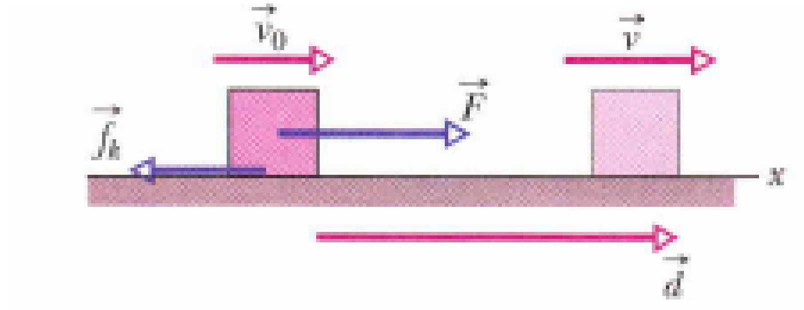
**No friction:**

For a <u>single particle</u>: $w = \Delta K$

For a <u>system of particles</u>: $w = \Delta K + \Delta U = \Delta E_{mec}$



**Friction involved:**




**For the block only:**

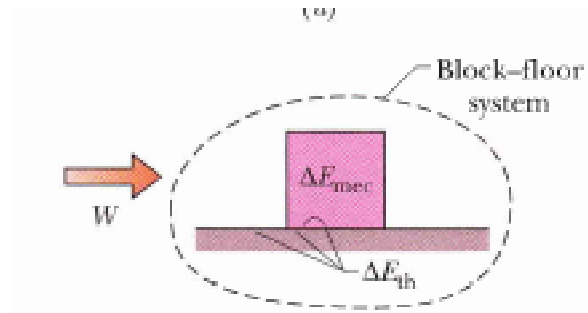
$$F - f_k = ma.$$

$$v^2 = v_0^2 + 2ad.$$

$\longrightarrow$ $Fd = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + f_k d$ $\longrightarrow$ $Fd = \Delta K + f_k d.$ **(1)**

**For the system of block and floor:**

$\Delta K \longrightarrow \Delta E_{mec}$ **(2)**




**Experiment shows:** $\Delta E_{th} = f_k d$ (increase in thermal energy by sliding). **(3)**

**(1),(2),(3)** $\longrightarrow$ $Fd = \Delta E_{mec} + \Delta E_{th}.$ $\longrightarrow$ $W = \Delta E_{mec} + \Delta E_{th}$ (work done on system, friction involved)

# Conservation of energy:

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}},$$

**Any change in mechanical energy of the system.**

**Any change in thermal energy of the system.**

**any change in any other type of internal energy of the system**

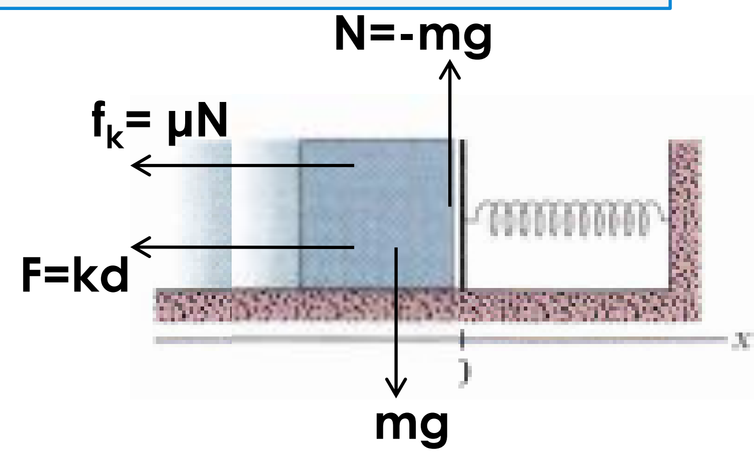
# Isolated system:

**In an isolated system:** **$w=\Delta E=0$**

$$W=\Delta E=\Delta E_{mec}+\Delta E_{th}+\Delta E_{int},$$

$$E_{mec,2}=E_{mec,1}-\Delta E_{th}-\Delta E_{int}.$$

A block of mass m : 2.5 kg slides head on into a spring of spring constant k : 320 N/m. When the block stops, it has compressed the spring by 7.5 cm. The coefficient of kinetic friction between block and floor is 0.25.




While the block is in contact with the spring and being brought to rest, what are (a) the work done by the spring force and (b) the increase in thermal energy of the block-floor system? (c) What is the block's speed just as it reaches the spring?

---

The work done by the spring force : $w=-\frac{1}{2}kx^2\,|^{x=d} =-1/2kd^2$

$w=-\frac{1}{2}(320)(0.075)^2$

$\boxed{w=-0.9\ j}$

---

the increase in thermal energy of the block-floor system:

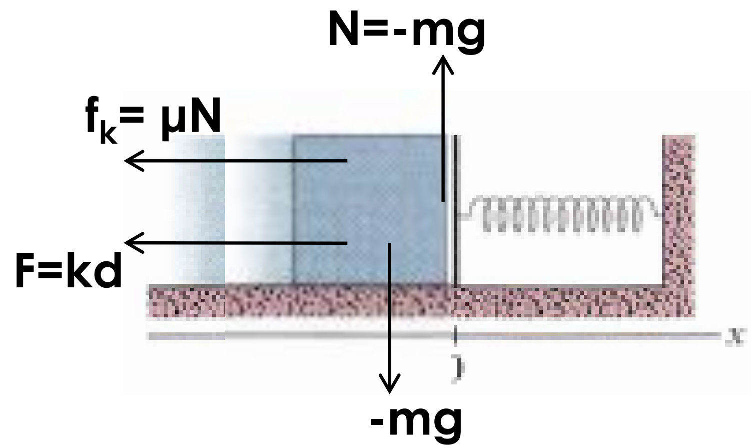
$E_{the}=f_k x\ \ |^{x=d}=f_k d=-\mu m(g)d$

$E_{the}=(0.25)(2.5)(10)(0.075)$

$\boxed{E_{the}=0.46875\ j}$

$$E_{mec,2} = E_{mec,1} - \Delta E_{th} - \Delta E_{int}.$$

$E_{the}$=0.46875 j




The block's speed just as it reaches the spring:

$E_1 = \frac{1}{2} m v_1^2$

$E_2 = \frac{1}{2} kd^2 - \mu mgd =$

In the isolated system of floor- block-spring:

$E_1 = E_2$

$\frac{1}{2} m v_1^2 = \frac{1}{2} kd^2 - \mu mgd$

$\frac{1}{2} m v_1^2 = 160*(0.075)^2 + 0.4687$

$= 0.9 + 47 = 1.37$

$v_1 = 1.1$ m/s